Interregional Negotiations and Strategic Delegation under Government Subsidy Schemes*

Ryusuke Shinohara[†]
Hosei University

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Abstract

We examine the *strategic delegation* problem in the context of interregional negotiations under the subsidy policies of a central government. It is well known that when such negotiations are delegated to representatives, each region in a country elects its representative strategically, resulting in inefficient negotiation outcomes. This study focuses on a common subsidy policy called a *cost-matching grant* to examine whether an optimal grant exists that restores the efficiency of negotiation outcomes. Our results show that the central government obtains this optimal grant if the manipulability of the negotiation breakdown outcome is sufficiently weak. The strength of the manipulability is important because introducing a grant generates a new kind of manipulation of negotiation breakdown outcomes. However, when a new representative is elected after a negotiation breaks down, the new manipulability is negated. Hence, the central government always obtains the optimal cost-matching grant.

Keywords Strategic delegation; Median voter theorem; Cost-matching grant; Lindahl price; Nash bargaining.
JEL classifications D62, D72, H41, H77.

1 Introduction

We examine the *strategic delegation problem* in the context of interregional negotiations under a government subsidy policy. Prior studies have found that in the absence of a government subsidy, negotiations are distorted by strategic delegation, such that efficiency is not achieved. Here, we examine whether the strategic delegation problem is mitigated or worsened under a government subsidy policy.

In the real world, although decisions on public projects with spillovers may be delegated to the regional level, the central government does not directly implement the projects. Instead, the relevant regions receive subsidies and cooperate voluntarily to carry out these projects. For instance, to maintain an international river (e.g., to reduce pollution and prevent floods), the countries in

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[†]Department of Economics, Hosei University, 4342 Aihara-machi, Machida-shi, Tokyo, 194-0298, JAPAN. E-mail: ryusukes@hosei.ac.jp.

the river basin may establish joint projects.[1] At the same time, a supranational political organization (e.g., the European Union (EU)) may offer grant schemes to assist with cooperation between relevant countries. For example, Belgium, France, Germany, Luxembourg, and the Netherlands established a joint program to reduce the problems caused by high-water and flooding along the Rhine and Meuse Rivers. The program was funded as the "Interreg Rhine Meuse Activities" (INTERREG IIc) by the European Commission.[2] In addition, Germany and the Netherlands established the sustainable development of floodplains (SDF) project along the Rhine River, funded by the European INTERREG IIIb program (Nijland, 2005).[3] Indirect control of a project through central government subsidies is characteristic of federal countries and political unions such as the EU, because authority is sometimes delegated to the state level.[4]

We build a game-theoretic model of the aforementioned situation in the context of strategic delegation; that is, the central government does not directly implement a project, and project decisions are assigned to the relevant regions. The model contains a country comprising two regions (Regions A and B). The regions have asymmetric roles. Only region A undertakes a public project that benefits both regions, and region B makes monetary transfers to region A as compensation. This captures the relation between upstream and downstream countries along an international river. Representatives are elected from among the residents through majority voting in each region. These representatives then negotiate the project level and transfers. If the negotiation breaks down, region A's representative independently decides the project level, and region B free rides. Prior to the election of the representatives and the negotiation, an upper-tier government of the regions (hereafter, the central government) establishes a *cost-matching grant* for the project in region A. Under the grant, a certain rate of region A's project cost is subsidized by the central government, and the subsidy is financed by a tax on region B.

It is well known that negotiation plays the role of internalizing an externality between the parties involved and, thus, improves allocative efficiency, according to the Coase theorem (Coase, 1960). However, when a representative negotiates on behalf of a party, the negotiation does not work sufficiently to achieve efficiency because the representative is elected strategically by the party to improve its bargaining position. Consequently, the bargaining outcome is Pareto inefficient and, in some cases, even Pareto-inferior to the outcome without negotiations. This is known as the strategic delegation problem (e.g., Segendorff, 1998; Besley and Coate, 2003; Buchholz et al., 2005; Dur and Roelfsema, 2005; Loeper, 2017; Cheikbossian, 2016). In our situation of an asymmetric role between regions, it has been shown that *without a central government subsidy policy*, the project does not reach the first-best efficient level, even through negotiation (e.g., Gradstein, 2004; Rota-Graziosi, 2009; Loeper, 2015; Shinohara, 2018). Thus, we examine the effects of strategic delegation on project efficiency in the context of government subsidy policies.

Our main finding is the possibility of achieving efficiency using a cost-matching grant, as summarized in Theorem 1. Theorem 1 states that a grant achieves efficiency if and only if (i) the cost-matching rate is based on the *Lindahl price*, and (ii) the manipulability of the negotiation breakdown outcome is sufficiently weak. The theorem shows an interesting property that the central government can obtain an optimal grant, conditional on the strength of the manipulability of the breakdown outcome. In our model, the project level is decided independently by region A's representative when the negotiation breaks down. Thus, the breakdown level can be manipulated by the choice of region A's representative. As we discuss in Section 3.2.2, introducing a grant gener-

[1] Although we use the terms "country" and "regions," our model can be applied to the relation between a supranational organization and its member countries.

[2] The EU financed 32% of the total investment, or ECU 134.118 million.
Data source: https://ec.europa.eu/regional_policy/archive/reg_prog/po/prog_663.htm

[3] The EU finances 50% of the total investment, or EUR 17.749 million.
Data Source: https://keep.eu/projects/1244/

[4] There is similar evidence in Japan, where some river maintenance projects are delegated to the local level. In this case, an upstream government will negotiate the cost-share with the downstream governments. At the same time, the central government subsidizes part of the upstream government's project cost; see Kobayashi and Ishida (2012) for details.

ates a new kind of manipulation of the negotiation breakdown outcomes. Therefore, the existence of an optimal grant is linked to the manipulability of the breakdown outcome. Prior studies have discussed how this manipulability is a major source of strategic delegation (e.g., Segendorff, 1998; Gradstein, 2004). However, this differs from our new manipulability.

We also show that the manipulability of the breakdown outcome does not matter if a new representative who only decides the breakdown outcome is elected after a negotiation breaks down. Thus, the election after the negotiation breaks down separates the responsibility for the negotiation and that for the decision of the breakdown outcome. By this separation, the cost-matching rate based on the Lindahl price always leads to the first-best efficient level (see Proposition 3). We also consider the cost-matching grant without the Lindahl price. As discussed in Section 3.4, if the grant is not based on the Lindahl price, then the equilibrium outcome with negotiations may be Pareto-inferior to that without negotiations. This indicates that a nonoptimal grant may make the strategic delegation more serious in combination with the negotiation.

Our theoretical analysis includes policy implications on combinations of policy instruments and voluntary negotiations. Several theoretical studies have shown that government interventions using subsidies and taxes may improve the efficiency of negotiation outcomes when the negotiation itself does not reach the first-best efficient level (in situations different to ours) (e.g., Lülfesmann, 2002; Lüelfesmann et al., 2015; Rosenkranz and Schmitz, 2007; and MacKenzie and Ohndorf, 2016). For the strategic delegation problem, we observe similar phenomena, *conditional* on the manipulability of the breakdown outcome. Thus, for the government, setting the cost-matching rate appropriately and coordinating the degree of the breakdown manipulability are important to restoring efficiency. Setting the rate using the Lindahl price is important because introducing a cost-matching grant without the Lindahl price may harm the efficiency of the negotiation outcome. In addition, our findings suggest that an institutional arrangement that employs a re-election after a negotiation breaks down plays a role in coordinating the manipulability of the breakdown outcome and enhancing the effectiveness of the grant.

Segendorff (1998) and Gradstein (2004) discuss the relationship between the role of a strategic delegation and the manipulability of a breakdown outcome. They show that if a representative both negotiates and decides the breakdown outcome, then the outcome achieved through an interregional negotiation may be Pareto-inferior, even to the allocation in the absence of negotiation. On the other hand, if the representatives are separated, then a negotiated outcome is Pareto-superior to that without a negotiation. Thus, the welfare-enhancing property of a negotiation depends on separating the authority for the negotiation and that for the breakdown outcome. In addition, our results show that this separation influences whether a government subsidy policy reaches the first-best efficient level.

Eckert (2003) and Buchholz et al. (2013) examine the delegation behavior of federal countries in the context of providing international public goods. Buchholz et al. (2013) show that the efficiency loss of the negotiation outcome due to the delegation can be mitigated by introducing a fiscal instrument, including a cost-matching grant, between a public good provider and the beneficiary of the good in each country in the federation. However, their models differ from ours in that they do not include selecting a representative using an election. In the literature on strategic delegation in an election, Dur and Roelfsema (2005) show that a central government subsidy policy, which differs from a cost-matching grant, eliminates the incentive for a strategic delegation. An important difference between our work and that of Dur and Roelfsema (2005) lies in the nature of the negotiation. The interregional negotiation of Dur and Roelfsema (2005) is “centralized” (i.e., legislative bargaining), in that no region can provide public goods independently if a negotiation breaks down. In contrast, our negotiation is “decentralized,” in that the regions act freely and independently after a negotiation breaks down. In the centralized negotiation of Dur and Roelfsema (2005), the manipulation of the breakdown outcome does not matter.

The remainder of this paper is organized as follows. Section 2 introduces the model, and Section 3 provides our analysis. Section 4 discusses extensions to our basic model. Lastly, Section 5 concludes the paper. All proofs are collected in the appendix.

2 The model

Consider two regions (regions A and B) within a country. Each region has a regional government. The government in region A undertakes a local public project $x \in \mathbb{R}_+$ that benefits both regions A and B, owing to the spillover effect of the project. The cost function of the public project x is given as $c : \mathbb{R}_+ \to \mathbb{R}_+$, such that $c(0) = 0$, $c'(x) > 0$, $c''(x) > 0$, $\lim_{x\to 0} c'(x) = 0$, and $\lim_{x\to\infty} c'(x) = \infty$. While region A incurs the cost of the project, region B transfers money to region A, denoted by T, as compensation.

Each region is populated with individuals. Residents in region A are symmetrically distributed over an interval $\mathcal{A} \equiv [\underline{a}, \bar{a}] \subset \mathbb{R}_+$, where $\bar{a} - \underline{a} = n_A > 0$. The population density function on $\mathcal{A}$ is denoted by f^A, such that region A's mean $\int_{a\in\mathcal{A}} af^A(a)\mathrm{d}a$ is denoted by a_M. Similarly, residents in region B are symmetrically distributed over $\mathcal{B} \equiv [\underline{b}, \bar{b}] \subset \mathbb{R}_+$, where $\bar{b} - \underline{b} = n_B > 0$, according to region B's population density function f^B, such that region B's mean $\int_{b\in\mathcal{B}} bf^B(b)\mathrm{d}b$ is denoted by b_M. Then, $a \in \mathcal{A}$ and $b \in \mathcal{B}$ represent the regions' respective tastes for the public project, which we explain in detail later. We do not impose a relative relation between $\mathcal{A}$ and $\mathcal{B}$. Because the population distributions of both regions are symmetric, their means and medians coincide.[5] In the basic model, we assume that the population is immobile across the regions.[6]

We assume that, in order to finance the project cost $c(x)$, region A's government taxes all residents equally. Each of region A's residents pays $c(x)/n_A$ to the regional government, where the population in region A is denoted by n_A.

The transfers from region B to region A are assumed to be shared equally by region B's residents through a tax by region B's government. Here, let T be the per-capita transfer of region B; that is, every resident in region B pays T to region B's government. The total amount of transfers is n_BT, which is assumed to be distributed equally to all residents in region A; that is, every resident in region A receives n_BT/n_A from the distribution.

There is an upper-tier government, called the central government, that provides a *cost-matching grant* to region A. Under this grant, if region A's government produces x units of the project, then the central government provides a subsidy of $(1-\gamma)c(x)$ to region A's government, where $\gamma \in [0, 1]$ is the cost-matching rate. We assume that (i) region A's government distributes $(1 - \gamma)c(x)$ to all residents in region A equally, and (ii) the subsidy $(1 - \gamma)c(x)$ is financed by an equal tax on all residents in region B by the central government. Hence, region A's *net* share of the project cost is $\gamma c(x)/n_A$, and region B's net share is $(1 - \gamma)c(x)/n_B$. The central government decides the value of γ in advance of a regional election and an interregional negotiation. Clearly, $\gamma = 1$ corresponds to a situation of "no cost-matching grant."

Given the regional and central government schemes, we assume that region A's resident, whose taste is $a \in \mathcal{A}$, has the utility function

$$U(x, T; a) = u(x, a) + \frac{I_A}{n_A} - \frac{\gamma}{n_A}c(x) + \frac{n_B}{n_A}T.$$

Here, $u(x, a)$ represents the gross benefit from the public project earned by a, I_A represents the total income of region A, I_A/n_A is the per-capita income of this region, and $-(\gamma/n_A)c(x) + (n_B/n_A)T$ represents the net payment from each resident in region A. Similarly, we assume that region B's resident $b \in \mathcal{B}$ has the utility function

$$V(x, T; b) = v(x, b) + \frac{I_B}{n_B} - \frac{1-\gamma}{n_B}c(x) - T,$$

[5] In our Theorem 1, the symmetry of the population distributions is essential for a "self-representation equilibrium" to achieve efficiency. In Section 3.5, we discuss the case in which the population distribution is asymmetric.

[6] In Section 4.3, we discuss the stability of the population distributions when the population is mobile.

where $\upsilon(x, b)$ is the gross benefit from the project earned by b, I_B is the total income of region B, I_B/n_B is the per-capita income of this region, and $-(1-\gamma)c(x)/n_B - T$ is the resident's payment. We assume that I_A and I_B are large enough to meet the tax payments.[7] We further assume that $u(x, a) = a\mu(x)$, where $\mu(0) = 0$, $\mu'(x) > 0$, and $\mu''(x) \leq 0$, and $\upsilon(x, b) = b\nu(x)$, where $\nu(0) = 0$, $\nu'(x) > 0$, and $\nu''(x) \leq 0$. Under this assumption,

$$u_x(x, a) \equiv \frac{\partial u(x, a)}{\partial x} > 0,\ u_{xx}(x, a) \equiv \frac{\partial^2 u(x, a)}{\partial x^2} \leq 0,\ u_a(x, a) \equiv \frac{\partial u(x, a)}{\partial a} > 0,$$
$$\text{and } u_{xa}(x, a) \equiv \frac{\partial^2 u(x, a)}{\partial a \partial x} = u_{ax}(x, a) \equiv \frac{\partial^2 u(x, a)}{\partial x \partial a} > 0.$$

Similarly,

$$\upsilon_x(x, b) \equiv \frac{\partial \upsilon(x, b)}{\partial x} > 0,\ \upsilon_{xx}(x, b) \equiv \frac{\partial^2 \upsilon(x, b)}{\partial x^2} \leq 0,\ \upsilon_b(x, b) \equiv \frac{\partial \upsilon(x, b)}{\partial b} > 0,$$
$$\text{and } \upsilon_{xb}(x, b) \equiv \frac{\partial^2 \upsilon(x, b)}{\partial b \partial x} = \upsilon_{bx}(x, b) \equiv \frac{\partial^2 \upsilon(x, b)}{\partial x \partial b} > 0.$$

Given the cost-matching grant, regions A and B negotiate the project level and the monetary transfers between the two regions. Formally, we consider a three-stage game with complete information. The sequence of the game is as follows:

Stage 0 The central government sets the cost-matching parameter $\gamma \in [0, 1]$.

Stage 1 After observing γ, each region selects a regional representative through majority voting. All residents are eligible to be the region's representative. Each resident is assumed to vote for her optimal candidate, anticipating the outcomes of subsequent stages.

Stage 2 The representatives of regions A and B negotiate over the levels of x and T, according to their interests. If they reach an agreement on these levels, then it is executed. If the negotiation breaks down, then region A's representative independently decides the project level x, and there are no interregional transfers (namely, $T = 0$). We analyze the negotiation in Stage 2 with the asymmetric *Nash bargaining solution*. This independent decision constitutes a breakdown outcome.

For a model with a unidirectional externality, the game, consisting of an election and a negotiation stage, has been examined by Gradstein (2004), Rota-Graziosi (2009), Loeper (2015), and Shinohara (2018). We include an additional stage that involves the central government (Stage 0), prior to the election and negotiation stages. In our model, given the cost-matching rate determined by the central government, the two regions negotiate through their representatives. Models of negotiations, given a government tax and a subsidy policy, have been investigated in Coasean situations (see Lülfesmann, 2002; Rosenkranz and Schmitz, 2007; Lüelfesmann et al., 2015; MacKenzie and Ohdorf, 2016). The election and negotiation, given the central government policy, can be observed when the cost-matching rate for some interregional public projects is set by law, and is independent of the negotiation results.[8]

With regard to the assumptions on the functional forms, our utility and cost functions are more general than those in previous studies. Gradstein (2004), Rota-Graziosi (2009), and Shinohara (2018) assume $\mu(x) = \nu(x) = x$ and $c(x) = x^2/2$. Loeper (2015, 2017) uses a different benefit function. Despite the generality, we show in Proposition 1 that a strategic delegation occurs.

[7] This assumption seems standard in studies on strategic delegation because a similar assumption is made in related studies, such as Besley and Coate (2003), Dur and Roelfsema (2005), and Cheikbossian (2016). Nevertheless, in Section 4.1, we discuss the case in which the budget feasible conditions matter.

[8] For example, in Japan, the cost-matching rate for some river maintenance projects is specified in the River Act. See Kobayashi and Ishida (2012).

3 Analysis

3.1 Efficient projects

We first identify the efficient level of a project. Because we assume a symmetric population distribution on $\mathcal{A}$ and $\mathcal{B}$, the total surplus of the overall economy is given by

$$n_A \int_{a\in\mathcal{A}} u(x,a) f^A(a)\mathrm{d}a + n_B \int_{b\in\mathcal{B}} v(x,b) f^B(b)\mathrm{d}b + I_A + I_B - c(x) = n_A a_M \mu(x) + n_B b_M \nu(x) + I_A + I_B - c(x).$$

Hence, the level of the project that maximizes the total surplus, denoted by x^E, satisfies

$$n_A a_M \mu'(x^E) + n_B b_M \nu'(x^E) = c'(x^E). \tag{1}$$

3.2 The cost-matching grant and the efficiency

3.2.1 Analysis of Stage 2

We solve the games by backward induction.

Given that $a_R \in \mathcal{A}$ and $b_R \in \mathcal{B}$ are representatives appointed in Stage 1, we first analyze the negotiation in Stage 2. If this negotiation breaks down, then region A's representative a_R decides the level of x independently, such that she maximizes her utility $U(x,0;a_R) = u(x,a_R) - (\gamma/n_A)c(x) + I_A/n_A$. If we denote the maximizer of $U(x,0;a_R)$ by x^A, then $u_x(x^A,a_R) = (\gamma/n_A)c'(x^A)$ by the first-order condition. By this condition, x^A depends on a_R and γ. Hence, we can express $x^A = x^A(a_R,\gamma)$, such that

$$u_x(x^A(a_R,\gamma),a_R) = \frac{\gamma}{n_A}c'(x^A(a_R,\gamma)). \tag{2}$$

Obviously, $x^A(a_R,\gamma)$ is well defined by the assumptions $u(x,a_R)$ and $c(x)$.

The project level when the negotiation breaks down is manipulated through the choice of region A's representative. Here, $\partial x^A(a_R,\gamma)/\partial a_R$ measures the degree of this manipulability, which plays a key role in our main result. By (2), this is calculated as

$$\frac{\partial x^A(a_R,\gamma)}{\partial a_R} = -\frac{u_{xa}(x^A,a_R)}{u_{xx}(x^A,a_R) - \frac{\gamma}{n_A}c''(x^A)} > 0. \tag{3}$$

Anticipating the breakdown outcome, representatives a_R and b_R decide x and T through Nash bargaining: x and T maximize the Nash product function

$$\beta \ln\left[u(x,a_R) + \frac{n_B}{n_A}T + \frac{I_A}{n_A} - \frac{\gamma}{n_A}c(x) - U(x^A,0;a_R)\right]$$
$$+(1-\beta)\ln\left[v(x,b_R) + \frac{I_B}{n_B} - \frac{1-\gamma}{n_B}c(x) - T - V(x^A,0;b_R)\right],$$

where $x^A = x^A(a_R,\gamma)$, and $\beta \in [0,1]$ is the *bargaining power* of a_R.[9] The properties of the bargaining outcome are presented as follows:

Lemma 1 Let x^{nb} and T^{nb} be the maximizers of the Nash product.[10] Then,

$$n_A u_x(x^{nb},a_R) + n_B v_x(x^{nb},b_R) = c'(x^{nb}), \text{ or } n_A a_R \mu'(x^{nb}) + n_B b_R \nu'(x^{nb}) = c'(x^{nb}), \tag{4}$$

[9] The regional bargaining power is assumed to be independent of who is chosen as the representative.

[10] The superscript "*nb*" denotes "Nash bargaining."

and

$$\frac{n_B}{n_A}T^{nb} = \frac{\beta}{n_A}\left[n_A u(x^{nb}, a_R) + n_B v(x^{nb}, b_R) - c(x^{nb}) - \left(n_A u(x^A, a_R) + n_B v(x^A, b_R) - c(x^A)\right)\right]$$
$$- \left[u(x^{nb}, a_R) - \frac{\gamma}{n_A}c(x^{nb}) - \left(u(x^A, a_R) - \frac{\gamma}{n_A}c(x^A)\right)\right]. \tag{5}$$

We denote $x^{nb} \equiv x^{nb}(a_R, b_R)$ and $T^{nb} \equiv T^{nb}(a_R, b_R)$, because x^{nb} and T^{nb} depend on the representatives' preferences (a_R, b_R), by (4) and (5).

From (4), we learn that the Nash bargaining determines the project level x^{nb} that maximizes $n_A u(x, a_R) + n_B v(x, b_R) - c(x)$. This can be interpreted as a *virtual total surplus* of the entire economy, as if every resident of region A (region B) has the same benefit function as its representative a_R (b_R, respectively). From (5), region A never makes transfers to region B because T^{nb} is nonnegative. T^{nb} is nonnegative because the term in brackets in the first line of (5) is the change of the virtual total surplus from the negotiation. This term is nonnegative because, from (4), x^{nb} maximizes $n_A u(x, a_R) + n_B v(x, b_R) - c(x)$. The term in brackets in the second line is a_R's individual surplus from the negotiation, which is nonpositive, by (2).

As the result of the Nash bargaining, each resident $a \in \mathcal{A}$ receives the payoff

$$\begin{aligned} U(x^{nb}, T^{nb}; a) &= u(x^{nb}, a) + \frac{I_A}{n_A} - \frac{\gamma}{n_A}c(x^{nb}) + \frac{n_B}{n_A}T^{nb} \\ &= u(x^{nb}, a) - u(x^{nb}, a_R) + u(x^A, a_R) - \frac{\gamma}{n_A}c(x^A) + \frac{I_A}{n_A} \\ &\quad + \frac{\beta}{n_A}\left[n_A u(x^{nb}, a_R) + n_B v(x^{nb}, b_R) - c(x^{nb}) - \left(n_A u(x^A, a_R) + n_B v(x^A, b_R) - c(x^A)\right)\right], \end{aligned} \tag{6}$$

and each resident $b \in \mathcal{B}$ receives the payoff[11]

$$\begin{aligned} V(x^{nb}, T^{nb}; b) &= v(x^{nb}, b) + \frac{I_B}{n_B} - \frac{1-\gamma}{n_B}c(x^{nb}) - T^{nb} \\ &= v(x^{nb}, b) - v(x^{nb}, b_R) + v(x^A, b_R) - \frac{1-\gamma}{n_B}c(x^A) + \frac{I_B}{n_B} \\ &\quad + \frac{1-\beta}{n_B}\left[n_A u(x^{nb}, a_R) + n_B v(x^{nb}, b_R) - c(x^{nb}) - \left(n_A u(x^A, a_R) + n_B v(x^A, b_R) - c(x^A)\right)\right]. \end{aligned} \tag{7}$$

In these payoff functions, the terms between the square brackets represent the virtual total surplus relative to the breakdown of the negotiation. According to the Nash bargaining solution, representative a_R receives the payoff at the breakdown of the negotiation (i.e., $u(x^A, a_R) - (\gamma/n_A)c(x^A)$) plus the surplus distribution in proportion to her bargaining power and region A's population. In addition, the difference between the project benefits of region A's residents and those of the representative (i.e., $u(x^{nb}, a) - u(x^{nb}, a_R)$) is included in the payoff function for all other residents a in region A. The same interpretation applies to the payoff functions of region B's residents.

3.2.2 Analysis of Stage 1

We now analyze Stage 1. First, note that the above payoff functions $U(x^{nb}, T^{nb}; a)$ and $V(x^{nb}, T^{nb}; b)$ satisfy the *single-crossing condition* of Gans and Smart (1996), which is shown in the appendix.

[11] Note that from (5), we have

$$T^{nb} = v(x^{nb}, b_R) - \frac{1-\gamma}{n_B}c(x^{nb}) - v(x^A, b_R) + \frac{1-\gamma}{n_B}c(x^A)$$
$$- \frac{1-\beta}{n_B}\left[n_A u(x^{nb}, a_R) + n_B v(x^{nb}, b_R) - c(x^{nb}) - (n_A u(x^A, a_R) + n_B v(x^A, b_R) - c(x^A))\right].$$

Hence, we regard the median resident in a region as being pivotal in the election in Stage 1. Anticipating the outcome in Stage 2, the median residents a_M and b_M choose their representatives (a_R and b_R, respectively) by maximizing $U(x^{nb}, T^{nb}; a_M)$ and $V(x^{nb}, T^{nb}; b_M)$, respectively. We examine the pure-strategy Nash equilibria of the game in Stage 1 by the median residents. By the definition of pure-strategy Nash equilibria, $(a_R^*, b_R^*) \in \mathcal{A} \times \mathcal{B}$ is a Nash equilibrium of this Stage 1 game if a_R^* and b_R^* are mutually best responses. That is, (a_R^*, b_R^*) is a solution of the first-order conditions,

$$\frac{\partial U(x^{nb}(a_R^*, b_R^*), T^{nb}(a_R^*, b_R^*); a_M)}{\partial a_R} = 0 \text{ and } \frac{\partial V(x^{nb}(a_R^*, b_R^*), T^{nb}(a_R^*, b_R^*); b_M)}{\partial b_R} = 0, \tag{8}$$

and the second-order conditions are satisfied.

In order to understand the median residents' incentives to choose their representatives, it is helpful to see the first derivatives of their payoff functions.

Lemma 2 It follows that

$$\begin{aligned} \frac{\partial\, U(x^{nb}, T^{nb}; a_M)}{\partial\, a_R} =& \frac{\partial x^{nb}}{\partial a_R} \left[u_x(x^{nb}, a_M) - u_x(x^{nb}, a_R)\right] \\ & - \left(\frac{\beta}{n_A}\right) \frac{\partial x^A}{\partial a_R} \left[n_A u_x(x^A, a_R) + n_B v_x(x^A, b_R) - c'(x^A))\right]' \\ & -(1-\beta)\left[u_a(x^{nb}, a_R) - u_a(x^A, a_R)\right] \end{aligned} \tag{9}$$

and

$$\frac{\partial\, V(x^{nb}, T^{nb}; b_M)}{\partial\, b_R} = \frac{\partial x^{nb}}{\partial b_R}[v_x(x^{nb}, b_M) - v_x(x^{nb}, b_R)] - \beta[v_b(x^{nb}, b_R) - v_b(x^A, b_R)]. \tag{10}$$

Region A's median resident can influence x^{nb}, T^{nb}, and x^A through the choice of a_R. The effects on $U(x^{nb}, T^{nb}; a_M)$ caused by the changes of these three elements are summarized in (9). In (9), the first line summarizes the effect of a change in x^{nb}, and the second line shows the effect of a change in x^A. The third line shows the "cross" effect of changes in the representative's preferences and in the project level. This comes from the effect on T^{nb} of a change in the benefit to region A's representative, $u(\cdot, a_R)$. Region A's median resident balances the three effects to choose the representative for region A.

In contrast, Region B's median resident can influence x^{nb} and T^{nb}, but not x^A, through the choice of b_R. Thus, we do not observe an effect from a change in x^A in (10), in contrast to (9). From (10), we observe that region B's median resident balances two effects.

Comparing (1) and (4), we find that the project is undertaken at the efficient level if the median resident votes for herself in every region: that is, $(a_R^*, b_R^*) = (a_M, b_M)$. We call the equilibrium with $(a_R^*, b_R^*) = (a_M, b_M)$ a *self-representation* equilibrium. Hereafter, focusing on the existence of the self-representation equilibrium, we examine whether the project is carried out efficiently with a cost-matching grant.[12]

First, as a reference point for the analysis, we show that, without the cost-matching grant, the median residents appoint residents whose preferences for the project are weaker than their own; see Proposition 1. Thus, the self-representation is impossible.

Proposition 1 *Suppose $\gamma = 1$. Then, the equilibrium representatives (a_R^*, b_R^*) must satisfy $a_R^* < a_M$ and $b_R^* \leq b_M$ with equality if $\beta = 0$. Hence, the equilibrium project level is below the efficient level.*

[12]Under our general utility and cost functions, there might be a case where the efficient project is achieved even if $(a_R^*, b_R^*) \neq (a_M, b_M)$. However, we consider it reasonable to focus on the self-representation, because this leads to an efficient project for *all* functions. Furthermore, in some cases (see Example 1), the self-representation is the only way to achieve the efficient project (see Footnote 16).

This was established by Gradstein (2004) under a special case of $u(x,a) = ax$, $v(x,b) = bx$, $c(x) = x^2/2$, and $\beta = 1$ (see Section 3.1 in Gradstein (2004)).

Now, we examine whether there is a cost-matching rate γ under which this self-representation behavior is supported in a Nash equilibrium of an induced Stage 1 game. Theorem 1 establishes a necessary and sufficient condition for the rate γ and the degree of manipulability of x^A under which the efficient project is achieved in equilibrium.

Theorem 1 *Let γ^* be a cost-matching rate such that*

$$\gamma^* \equiv \frac{n_A u_x(x^{nb}(a_M, b_M), a_M)}{n_A u_x(x^{nb}(a_M, b_M), a_M) + n_B v_x(x^{nb}(a_M, b_M), b_M)},$$

which is a "Lindahl price" based on the preferences of region A's median resident at x^{nb}. Then,

(i) $(a_R, b_R) = (a_M, b_M)$ is a solution of the first-order conditions (8) *if and only if $\gamma = \gamma^*$.*

(ii) The second-order condition for maximization is satisfied at $(a_R, b_R) = (a_M, b_M)$ if and only if $\frac{\partial x^A(a_R,\gamma)}{\partial a_R}\Big|_{a_R=a_M,\gamma=\gamma^*}$ is sufficiently small.

Therefore, there exists a subgame-perfect equilibrium in which the median resident votes for herself in every region and the efficient public project is achieved if and only if $\gamma = \gamma^*$ and $\frac{\partial x^A(a_R,\gamma)}{\partial a_R}\Big|_{a_R=a_M,\gamma=\gamma^*}$ is sufficiently small.

As Lemma 3 clarifies, the cost-matching grant plays a role in coordinating a relative relation of the project level when the negotiation succeeds or fails, given that the median residents self-represent.

Lemma 3 $x^A(a_M, \gamma) \gtreqless x^{nb}(a_M, b_M)$ if $\gamma \lesseqgtr \gamma^*$.

Theorem 1 shows that coordinating the relative relation between the two levels of the project is crucial for the self-representation to be supported in equilibrium. The Lindahl price γ^* plays a role in eliminating the strategic effect due to the changes to x^{nb}, x^A, and the benefits $u(\cdot, a_R)$ and $v(\cdot, b_R)$ in (9) and (10). This is because, under the optimal grant γ^*, the negotiated level of the project and the level at a disagreement are equated: $x^{nb}(a_M, b_M) = x^A(a_M, \gamma^*)$. Thus, the negotiated level of the project can be achieved even if the negotiation breaks down and the surplus of the negotiation is zero. The strategic delegation occurs because the median residents try to strategically increase their distribution of the negotiation surplus. However, because the negotiation generates no surplus, the median residents no longer strategically choose their representatives under the optimal grant.

The condition of $\partial x^A(a_R, \gamma)/\partial a_R$ in Theorem 1 guarantees meeting the second-order condition for region A's median resident. We can show that the second-order condition for region B's median resident always holds (see Claim 2 in the appendix). However, as Example 1 shows, the second-order condition for region A's median resident does not always hold.

Example 1 Suppose $c(x) = x^2/2$, $u(x,a) = ax$, and $v(x,b) = bx$. Then, from (2),

$$x^A(a_R, \gamma) = \frac{n_A a_R}{\gamma},\ U(x^A, 0; a_R) = \frac{n_A a_R^2}{2\gamma},\ \text{and } V(x^A, 0; b_R) = \frac{2\gamma b_R a_R n_A n_B - (1-\gamma) a_R^2 n_A^2}{2\gamma^2 n_B}.$$

From Lemma 1, Nash bargaining achieves

$$x^{nb}(a_R, b_R) = n_A a_R + n_B b_R \ \text{ and } \ T^{nb} = \frac{(\beta+\gamma)(n_A a_R(\gamma-1) + n_B b_R \gamma)^2}{2\gamma^2 n_B}.$$

From (6), given the Nash bargaining outcome, each resident $a \in \mathcal{A}$ obtains

$$U(x^{nb}, T^{nb}; a) = a\,(n_A a_R + n_B b_R) - \frac{\gamma}{2n_A}\,(a_R + b_R)^2 + \frac{(\beta+\gamma)\,(n_A a_R(\gamma-1) + n_B b_R \gamma)^2}{2\gamma^2 n_A} + \frac{I_A}{n_A}$$

and

$$\frac{\partial^2 U(x^{nb}, T^{nb}; a)}{\partial a_R^2} < 0 \iff n_A\left((\beta-2)\gamma^2 + (1-2\beta)\gamma + \beta\right) < 0$$
$$\iff 1 \geq \gamma > \underline{\Gamma}(\beta) \equiv \frac{1-2\beta+\sqrt{1+4\beta}}{2(2-\beta)}. \tag{11}$$

Hence, $U(x^{nb}, T^{nb}; a_M)$ is concave in a_R if γ is greater than the cutoff value $\underline{\Gamma}(\beta)$. Because $\underline{\Gamma}(\beta)$ is increasing in β, $\left(\underline{\Gamma}(\beta), 1\right]$ shrinks as β increases. Thus, when the surplus distribution to region A is large (i.e., β is large), a high cost-share for region A (i.e., a high γ) is necessary to mitigate the strategic behavior of the choice of region A's representatives and then for region A's second-order condition to be met. In addition, because $\underline{\Gamma}(0) = 1/2$, the second-order condition for region A's median resident holds only if $\gamma > 1/2$. Whether condition (11) depends only on β relies on the functional forms. Indeed, we observe that under the same linear benefit function, but with $c(x) = x^3/3$, the second-order condition for region A's median resident depends on the relative position between $\mathcal{A}$ and $\mathcal{B}$.[13]

In contrast, from (7), each $b \in \mathcal{B}$ obtains

$$V(x^{nb}, T^{nb}; b) = b(n_A a_R + n_B b_R) - \frac{1-\gamma}{2n_B}(n_A a_R + n_B b_R)^2 - \frac{(\beta+\gamma)\left(n_A a_R(\gamma-1) + n_B b_R \gamma\right)^2}{2\gamma^2 n_B} + \frac{I_B}{n_B}.$$

Here, $V(x^{nb}, T^{nb}; b_M)$ is always concave in b_R because

$$\frac{\partial^2 V(x^{nb}, T^{nb}; b_M)}{\partial b_R^2} = -1 - \beta < 0.$$

In summary, the second-order condition for region A's median resident holds at $\gamma = \gamma^*$ if and only if the Lindahl price γ^* is *sufficiently large*; that is, $\gamma^* = n_A a_M/(n_A a_M + n_B b_M) \in \left(\underline{\Gamma}(\beta), 1\right]$. Furthermore, $\gamma^* \geq \underline{\Gamma}(\beta)$ if and only if

$$\frac{n_A a_M}{n_B b_M} \geq \frac{1-2\beta+\sqrt{1+4\beta}}{3-\sqrt{1+4\beta}},$$

and the range of the right-hand side of the above inequality takes at least one, because[14]

$$1 \leq \frac{1-2\beta+\sqrt{1+4\beta}}{3-\sqrt{1+4\beta}} \leq \frac{\sqrt{5}-1}{3-\sqrt{5}}.$$

Thus, the second-order condition for region A's median resident holds when $\gamma = \gamma^*$ if $n_A a_M$ is sufficiently larger than $n_B b_M$.

We proceed with the analysis under the assumption of $\gamma \in \left(\underline{\Gamma}(\beta), 1\right]$.[15] We derive the equilibrium representatives (a_R^*, b_R^*) for the median residents by solving (8), such that

$$\begin{aligned} a_R^* &= -\frac{\gamma^2}{(\beta-2)\gamma^2 + (1-2\beta)\gamma + \beta} a_M - \frac{n_B \gamma(\beta\gamma - \beta - \gamma)}{((\beta-2)\gamma^2 + (1-2\beta)\gamma + \beta)n_A} b_R^*, \\ \text{and } b_R^* &= \frac{1}{1+\beta} b_M + \frac{\beta(1-\gamma)n_A}{(1+\beta)\gamma n_B} a_R^*. \end{aligned} \tag{12}$$

In (12), the first line is the best response of region A's median resident, and the second is that of region B's median resident. Note that the best response of region A's median resident shows a *strategic*

[13] See Section A of the online appendix.

[14] Note that $(1-2\beta+\sqrt{1+4\beta})/(3-\sqrt{1+4\beta})$ is increasing in β and takes 1 at $\beta = 0$.

[15] In Section 3.4.2, we discuss the case in which (11) does not hold.

substitute, because the coefficient of b_R^* is negative, by (11); in contrast, that of region B's median resident exhibits a *strategic complement*, because the coefficient of a_R^* is positive. Solving (12) yields

$$\begin{aligned} a_R^* &= \frac{\gamma((n_A a_M - n_B b_M)\gamma + \beta((n_A a_M + n_B b_M)\gamma - n_B b_M))}{(\beta+\gamma)(2\gamma-1)}, \\ b_R^* &= \frac{n_B b_M \gamma(2\gamma-1) + \beta(1-\gamma)((n_A a_M + n_B b_M)\gamma - b_M)}{(\beta+\gamma)(2\gamma-1)}, \\ \text{and } x^{nb}(a_R^*, b_R^*) &= n_A a_R^* + n_B b_R^* = \frac{(n_A a_M + n_B b_M)\gamma - n_B b_M}{2\gamma - 1}. \end{aligned} \tag{13}$$

Therefore, if $\gamma^* > \underline{\Gamma}(\beta)$ and $\gamma = \gamma^*$, then $(a_R^*, b_R^*) = (a_M, b_M)$ and $x^{nb} = n_A a_M + n_B b_M$.[16]

In Example 1, (11) shows that the second-order condition for region A's median resident is satisfied if γ is sufficiently large. This is consistent with the condition of $\partial x^A(a_R,\gamma)/\partial a_R$ in Theorem 1, because $\partial x^A(a_R,\gamma)/\partial a_R$ is decreasing with respect to γ in this example. Hence, if γ is sufficiently large, then $\partial x^A(a_R,\gamma)/\partial a_R$ is sufficiently small.[17] Thus, γ influences the strength of the manipulability of the breakdown outcome x^A through the choice of a_R.

We discuss how region A's median resident manipulates x^A, with and without the cost-matching grant. First, we consider the case of no cost-matching grant ($\gamma = 1$). In this case, an increase in a_R increases x^A, which, in turn, increases the disagreement payoff of region B's representative. Under Nash bargaining, this increase raises the bargaining surplus distributed to region B's representative. Thus, the transfers to region A, T^{nb}, decrease. Therefore, region A's median resident has an incentive to decrease a_R to improve her payoff through an increase in transfers, which leads to the equilibrium behavior in Proposition 1.

However, in addition to the incentive to decrease a_R, region A's median resident may have a strong incentive to increase a_R if the cost-matching grant is introduced ($\gamma < 1$). When $\gamma < 1$, the increase in x^A, which is induced by the increase in a_R, increases region B's cost burden $(1-\gamma)c(x^A)$, while his benefit $v(x^A, b_R)$ increases. When γ is sufficiently small (i.e., region B's cost burden is sufficiently large), the increase in the cost burden due to the change in a_R dominates the increase in his benefit. Then, the disagreement payoff of region B's representative decreases, leading to an increase in transfers to region A, which makes region A's median resident better off. The payoff function of region A's median resident may be *convex* in a_R if the effect of increasing region B's cost burden is much larger than the increase in the benefit to region B's representative. Later, in Section 3.4.2, we show the equilibrium in which the median resident of region A selects the resident with the highest valuation (i.e., $\bar{a}$) as region A's representative if γ is sufficiently small (see (i) of Result 1). This result supports that region A's median resident may drastically increase a_R in the presence of a cost-matching grant with sufficiently small γ.

Remark 1 In the proof of Theorem 1, we check that the second-order condition is satisfied *locally* at the self-representation $(a_R, b_R) = (a_M, b_M)$ and $\gamma = \gamma^*$. In order for the self-representation equilibrium to be a *global* solution, we need to examine whether the payoff functions of the median residents are concave in their representative characteristics. Proposition 2 shows that under some reasonable benefit and cost functions, which generalize those in Example 1, the payoff functions are concave in the representative parameters.

Proposition 2 *Suppose $u(x,a) = ax$, $v(x,b) = bx$, and $c(x) = x^\alpha/\alpha$, such that $\alpha \geq 2$. If $\partial x^A(a_R,\gamma)/\partial a_R$ is sufficiently small, then $U(x^{nb}, T^{nb}; a_M)$ is concave in a_R. In addition, $V(x^{nb}, T^{nb}; b_M)$ is always concave in b_R.*

The case of $\alpha \geq 2$ includes many cost functions. Hence, we conclude that, in many cases, $U(x^{nb}, T^{nb}; a_M)$ and $V(x^{nb}, T^{nb}; b_M)$ are concave in a_R and b_R, respectively.

[16] From (13), $x^{nb}(a_R^*, b_R^*) = x^E$ only if $\gamma = \gamma^*$, implying that $(a_R^*, b_R^*) = (a_M, b_M)$. Hence, $(a_R^*, b_R^*) = (a_M, b_M)$ is a necessary condition for $x^{nb}(a_R^*, b_R^*) = x^E$ in this example.

[17] This is also true in more general cases in which $u(x,a) = ax$, $v(x,b) = bx$, and $c(x) = x^\alpha/\alpha$ ($\alpha \geq 2$).

3.3 The re-election possibility

In the basic model, the same representative of region A negotiates with region B's representative and decides x^A if the negotiation breaks down. If the cost of a re-election is relatively low, region A may conduct a new election to choose another representative after the breakdown of the negotiation. We consider this case in this subsection. Region A can choose the same representative before and after the negotiation if the same resident is chosen in both elections. Hence, this model investigates whether region A commits to using the same representative. The new model is defined formally as follows.

Stages 0 and 1 are the same as those in the basic model. In Stage 2, the representatives $a_R \in \mathcal{A}$ and $b_R \in \mathcal{B}$ elected in Stage 1 negotiate (x, T). If they reach an agreement on (x, T), it is implemented, and the game ends. However, if this negotiation breaks down, the game moves to Stage 3. In Stage 3, a new representative $\tilde{a}_R \in \mathcal{A}$ is chosen through a new election with majority voting in Region A. This representative then chooses the project level after the negotiation breaks down.

With the possibility of a re-election, the selection of the negotiation representative can be separated from the selection of the decision maker of the breakdown outcome. In this case, the choice in Stage 1 has no effect on the project level when the negotiation breaks down; that is, $\partial x^A / \partial a_R = 0$. By this property, the second-order condition for region A's median resident always holds, and γ^* leads to the efficient project.

Proposition 3 *With the re-election after the negotiation breaks down, there is a subgame-perfect equilibrium in which the project is carried out efficiently if* $\gamma = \gamma^*$.

3.4 Discussion: The cost-matching rate other than γ^*

The Lindahl price depends on the information about the median residents' benefit functions. Thus, it might be difficult for the central government, which falls outside the two regions in our model, to acquire such information. Hence, it is important to discuss how introducing a cost-matching grant affects the bargaining outcome if the central government sets a cost-sharing rate other than the Lindahl price. Our discussion is based on Example 1.

3.4.1 The case of $\gamma \in \left(\underline{\Gamma}(\beta), 1\right]$

First, we consider the case in which γ is sufficiently large that it satisfies (11). As calculated previously, the project level is $x^{nb}(a_R^*, b_R^*)$ in (13) if the two regions negotiate. As a reference point, we derive the project level when the two regions do not negotiate. In this case, first, the two regions select their representatives through elections, and then region A's representative decides on the project level. We find that region A's median resident chooses herself as the representative, and sets $x^A(a_M, \gamma) = n_A a_M / \gamma$ in equilibrium.[18] Comparing the cases with and without the negotiation, we find that

$$\begin{aligned} x^E \geq x^A(a_M, \gamma) \geq x^{nb}(a_R^*, b_R^*) \ \text{ if } \gamma \geq \frac{n_A a_M}{n_A a_M + n_B b_M}, \\ \text{and } x^E \leq x^A(a_M, \gamma) \leq x^{nb}(a_R^*, b_R^*) \ \text{ if } \gamma \leq \frac{n_A a_M}{n_A a_M + n_B b_M}, \end{aligned} \tag{14}$$

where $x^E = n_A a_M + n_B b_M$.

Without the cost-matching grant (i.e., $\gamma = 1$), the project level through negotiation is the same as that without a negotiation; that is, $x^{nb} = x^A = n_A a_M$. On the one hand, the negotiation improves the total surplus of the overall economy because it internalizes the benefits from the project earned by the two regions. However, the strategic delegation has a counter effect, and the two effects offset each other. As a result, the project level is the same, with or without a negotiation.

[18] This can be verified in a similar way to the second paragraph of the proof of Proposition 3.

We conclude from (14) that the strategic delegation effect is worsened by the introduction of the cost-matching grant (i.e., $\gamma < 1$). From (14), the negotiation outcome is always Pareto-inferior to that without a negotiation, unless the cost-matching rate is the Lindahl price $n_A a_M/(n_A a_M + n_B b_M)$. This is because the level without bargaining is closer to the efficient level than is the level with bargaining. As discussed after Example 1, the introduction of the cost-matching grant affords region A's median resident a new way to manipulate the disagreement payoff to region B's representative. As such, the strategic delegation effect dominates the internalization effect of the negotiation. We have shown that introducing a nonoptimal cost-matching grant may detract from the welfare-improving property of the negotiation.

3.4.2 The case of $\gamma \notin \left(\underline{\Gamma}(\beta), 1\right]$

Second, we consider the case in which γ is sufficiently small that $\gamma \leq \underline{\Gamma}(\beta)$. In this case, region A's second-order condition is violated. To simplify the discussion, we examine the case of $\beta = 1/2$, $n_A = n_B = 1$, and $b_M = 1/2 \leq a_M$. Then, we have $\mathcal{A} = [\underline{a}, \bar{a}]$ and $\mathcal{B} = [0, 1]$, where $\underline{a} = a_M - 1/2$ and $\bar{a} = a_M + 1/2$. By (11), we need to consider γ, such that $0 < \gamma \leq \underline{\Gamma}(1/2) = 1/\sqrt{3} \approx 0.577$.

In this case, the best responses of the median residents and the equilibrium representatives are summarized as follows:

Result 1 *(i) Suppose $\gamma \in [0, a_M/(1 + a_M)]$. Then, the best response function of region A's median resident $a^*_R(b_R)$ is $a^*_R(b_R) = \bar{a}$, for all $b_R \in \mathcal{B}$, and the best response function of region B's median resident $b^*_R(a_R)$ ($a_R \in \mathcal{A}$) is the same as that in* (12)*:*

$$b^*_R(a_R) = \min\left\{1, \frac{1}{3} + \frac{a_R(1-\gamma)}{3\gamma}\right\}.$$

Thus, the equilibrium representatives are $a^*_R = \bar{a}$ and $b^*_R = b^*_R(\bar{a})$.
(ii) Suppose $\gamma \in \left(a_M/(1 + a_M), 1/\sqrt{3}\right]$. Then, the best response function of region A's median resident is

$$a^*_R(b_R) = \begin{cases} \bar{a} & \text{if } 0 \leq b_R \leq \frac{a_M(1-\gamma)}{\gamma} \\ \underline{a} & \text{if } 1 \geq b_R \geq \frac{a_M(1-\gamma)}{\gamma} \end{cases},$$

and the best response function of region B's median resident $b^*_R(a_R)$ ($a_R \in \mathcal{A}$) is the same as that in (i) above. There is no equilibrium.[19]

In Result 1, γ is "too small" in case (i), and is "intermediate" in case (ii). In case (i), region A's median resident selects the resident with the highest benefit as the regional representative. This is natural, because region A's cost-share of the project is low when γ is small, which induces region A's median resident to undertake the project at a high level. The project level in equilibrium is

$$x^{nb} = \bar{a} + b^*_R(\bar{a}) = a_M + \frac{1}{2} + b^*_R(\bar{a}),$$

which is greater than $x^E = a_M + 1/2$, because $b^*_R(\bar{a}) > 0$. Thus, the project is undertaken over the efficient level.

In case (ii), note that $a^*_R(b_R)$ is nonincreasing with respect to b_R. As we saw in (12) in Example 1, the best response function of region A's median resident is decreasing with respect to b_R. A somewhat weaker property of the strategic substitution is still observed in case (ii). However, in contrast to (12), region A's best response function is now discontinuous in the present case. This discontinuity leads to the nonexistence of the equilibrium.

In conclusion, setting γ sufficiently small may be used to increase the level of the project, but the level is not first-best efficient.

[19]The proof is given in Section B of the online appendix.

3.5 Discussion: On an asymmetric population distribution

We briefly discuss how our main findings change when the population is distributed asymmetrically. Under such distributions, $a_M \neq \tilde{a} \equiv \int_{a\in\mathcal{A}} af^A(a)\mathrm{d}a$ and $b_M \neq \tilde{b} \equiv \int_{b\in\mathcal{B}} bf^B(b)\mathrm{d}b$ (a_M and b_M are medians, and $\tilde{a}$ and $\tilde{b}$ are means).

Note that even under asymmetric population distributions, the median voter theorem holds in the basic models. Hence, the median resident is decisive in the choice of a representative through an election. Whether the payoff functions of the median residents are concave does not depend on the population distribution (see Proposition 2 and its proof). That is, whether the second-order conditions hold for the median residents does not depend on the population distribution. The discussion after Example 1 on the manipulability of region A's median resident through the grant also applies to cases with asymmetric population distributions.

Based on the functional forms in Example 1, we discuss the existence of an optimal rate γ that achieves the first-best efficiency. Under asymmetric population distributions, self-representation clearly no longer achieves efficiency. This is confirmed in Example 1, where $x^E = n_A\tilde{a} + n_B\tilde{b}$. Because $x^{nb}(a_R^*, b_R^*) = (\gamma(n_A a_M + n_B b_M) - n_B b_M)/(2\gamma - 1)$, we have that

$$x^{nb}(a_R^*, b_R^*) = n_A\tilde{a} + n_B\tilde{b} \iff \gamma = \gamma^{opt} \equiv \frac{(n_A\tilde{a} + n_B\tilde{b}) - n_B b_M}{2(n_A\tilde{a} + n_B\tilde{b}) - (n_A a_M + n_B b_M)}.$$

Thus, if $\gamma = \gamma^{opt} \in \left(\underline{\Gamma}(\beta), 1\right]$, then the equilibrium project level is efficient. The central government may determine the optimal cost-matching rate if the manipulability of the disagreement project is relatively weak.

In addition, we show that the introduction of γ^* (Theorem 1) is possibly more efficient than the case of no cost-matching grant, even under asymmetric population distributions. First, consider that the population distribution is *positively skewed* (i.e., $a_M < \tilde{a}$ and $b_M < \tilde{b}$). As we have already seen in Proposition 1, if there is no cost-matching grant ($\gamma = 1$), then the equilibrium representatives (a_R^*, b_R^*) satisfy $a_R^* < a_M$ and $b_R^* \leq b_M$. If a cost-matching grant based on the Lindahl price γ^* is introduced, self-representation is achieved in equilibrium. Because the population distribution is positively skewed and $x^{nb}(a_R, b_R)$ is increasing with respect to a_R and b_R, we have the relation $x^{nb}(a_R^*, b_R^*) < x^{nb}(a_M, b_M) < x^{nb}(\tilde{a}, \tilde{b}) = x^E$. That is, the equilibrium outcome is more efficient when the cost-matching grant with γ^* is introduced than it is when no cost-matching grant is introduced.

In contrast, if the population distribution is *negatively skewed* (i.e., $a_M > \tilde{a}$ and $b_M > \tilde{b}$), then the self-representation does not necessarily lead to a more efficient outcome than that in the case of no cost-matching grant. The discussion is based on Example 1. The equilibrium project level is $x^{nb}(a_R^*, b_R^*) = n_A a_M$ if no cost-matching grant is introduced, and $x^{nb}(a_M, b_M) = n_A a_M + n_B b_M$ if the cost-matching rate with γ^* is introduced. Under the negatively skewed distribution, the project level in equilibrium is below the efficient level if there is no cost-matching grant, and is over the efficient level if γ^* is introduced: $x^{nb}(a_R^*, b_R^*) < x^E < x^{nb}(a_M, b_M)$. Thus, self-representation does not always yield a more efficient outcome than that in the case of no cost-matching grant. The condition under which self-representation is more efficient than the case of no cost-matching grant is $|x^{nb}(a_R^*, b_R^*) - x^E| > |x^{nb}(a_M, b_M) - x^E|$, which is equivalent to

$$\frac{n_A}{n_B} < \frac{2\tilde{b} - b_M}{2(a_M - \tilde{a})}.$$

Because $a_M - \tilde{a} > 0$, by the negative skewness, there is a population pair (n_A, n_B), such that introducing γ^* is Pareto-superior to no cost-matching grant if the mean benefit of region B, $\tilde{b}$, is sufficiently close to the median, b_M, such that $b_M > \tilde{b} > b_M/2$.

4 Extensions

4.1 Budget balance condition

In the analysis in Section 3, we assumed that each region has enough income to meet any tax payment.[20] Here, we relax this assumption. We still assume that the provision of x^E in (1) is feasible in the economy: $c(x^E) < I_A + I_B$. We discuss the importance of (i) the controllability of the breakdown level of the project, and (ii) the unbiased bargaining powers of the regions for the project to be completed efficiently.

Concerning point (i), when the negotiation breaks down, region A's representative a_R chooses x^A to maximize his payoff $u(x^A, a_R) + (I_A/n_A) - (\gamma/n_A)c(x^A)$, subject to the budget conditions

$$\frac{\gamma}{n_A}c(x^A) \le \frac{I_A}{n_A} \text{ and } \frac{1-\gamma}{n_B}c(x^A) \le \frac{I_B}{n_B}. \tag{15}$$

The former (latter) is the budget condition for region A (region B, respectively). Because the cost of x^A is shared through the cost-matching grant of the central government, region B's budget condition constrains the decision of region A's representative. Then, x^A satisfies

$$\begin{aligned} &u_x(x^A, a_R) = \frac{\gamma}{n_A}c'(x^A) \text{ if } c(x^A) \le \min\left\{\frac{I_A}{\gamma}, \frac{I_B}{1-\gamma}\right\} \text{ and} \\ &c(x^A) = \min\left\{\frac{I_A}{\gamma}, \frac{I_B}{1-\gamma}\right\} \text{ otherwise.} \end{aligned} \tag{16}$$

Here, (16) shows that if the income distribution is too biased such that one of I_A/γ and $I_B/(1-\gamma)$ is too low, then x^A does not maximize representative a_R's surplus, $u(x, a_R) - (\gamma/n_A)c(x)$. After Theorem 1, we discussed that γ^* attains $x^A(a_M, \gamma^*) = x^{nb}(a_M, b_M)$, which prevents the strategic delegation. This is still possible if x^A, maximizing a_R's surplus, satisfies the budget feasibility in (15) at $\gamma = \gamma^*$. Otherwise, the breakdown project is lower than $x^{nb}(a_M, b_M)$. Hence, γ^* may not control the breakdown level of the project in the presence of the budget conditions. To restore this controllability of the breakdown level of the project, it may be helpful for the central government to redistribute the income between the regions.

We now discuss point (ii). As in Section 3, the Nash bargaining problem is formulated as

$$\begin{aligned} &\max_{x^{nb}, X_A^{nb}, X_B^{nb}} \; \beta \ln\left[u(x^{nb}, a_R) + X_A^{nb} - \bar{u}^A\right] + (1-\beta)\ln\left[v(x^{nb}, b_R) + X_B^{nb} - \bar{v}^B\right] \\ &\text{subject to } I_A + I_B = n_A X_A^{nb} + n_B X_B^{nb} + c(x^{nb}),\; X_A^{nb} \ge 0, \text{ and } X_B^{nb} \ge 0, \end{aligned}$$

where X_i^{nb} ($i = A, B$) represents the per-capita consumption of private goods of region i, and the first constraint represents the resource constraint in the economy. Furthermore, $\bar{u}^A$ and $\bar{v}^B$ are the breakdown payoffs to the representatives, such that

$$\bar{u}^A \equiv u(x^A, a_R) + \max\left\{0, \frac{I_A}{n_A} - \frac{\gamma}{n_A}c(x^A)\right\} \text{ and } \bar{v}^B \equiv v(x^A, b_R) + \max\left\{0, \frac{I_B}{n_B} - \frac{1-\gamma}{n_B}c(x^A)\right\}.$$

As in the main text, the Nash bargaining outcome depends on the types of the representatives. Then, we denote $x^{nb} = x^{nb}(a_R, b_R)$ and $X_i^{nb} = X_i^{nb}(a_R, b_R)$ ($i = A, B$).

By solving the Nash bargaining problem, we have that if $X_i^{nb}(a_R, b_R) > 0$, for all $i \in \{A, B\}$, then

$$X_A^{nb} = \frac{\beta}{n_A}\left(n_A u(x^{nb}, a_R) + n_B v(x^{nb}, b_R) + I_A + I_B - c(x^{nb}) - n_A\bar{u}^A - n_B\bar{v}^B\right) - \left(u(x^{nb}, a_R) - \bar{u}^A\right)$$

$$X_B^{nb} = \frac{1-\beta}{n_B}\left(n_A u(x^{nb}, a_R) + n_B v(x^{nb}, b_R) + I_A + I_B - c(x^{nb}) - n_A\bar{u}^A - n_B\bar{v}^B\right) - \left(v(x^{nb}, a_R) - \bar{v}^B\right),$$

[20] A detailed discussion is presented in Section C of the online appendix.

where $x^{nb} = x^{nb}(a_R, b_R)$ and $x^A = x^A(a_R, \gamma)$, and $\bar{u}^A$ and $\bar{v}^B$ are determined as above.[21]

By this result, we can say that the value of β determines whether these $X_i^{nb}(a_R, b_R)$ $(i = A, B)$ are positive. If β is sufficiently small, then the right-hand side of $X_A^{nb}(a_R, b_R)$ may be negative.[22] If β is sufficiently large, then the right-hand side of $X_B^{nb}(a_R, b_R)$ may be negative. Hence, $X_A^{nb}(a_R, b_R)$ and $X_B^{nb}(a_R, b_R)$ are both positive if and only if β is intermediate (the bargaining powers of the two regions are not biased). If β is too large or too small, then the private good consumption of one of the regions is zero.

In Section C of the online appendix, we show that when the bargaining powers of the regions are biased such that the private good consumption of one of the regions is zero, the project cannot be undertaken efficiently through self-representation. Therefore, when the budget conditions are introduced, an argument similar to that in Section 3 can be applied if the bargaining powers are nonbiased and the consumptions of the private good in both regions are positive.

4.2 A model of the endogenous choice of γ

In this study, we examine whether a central government obtains an optimal cost-matching rate that leads to the efficient undertaking of a project in the presence of interregional negotiations. Hence, an endogenous choice of γ has not been examined. In this subsection, we present an analysis based on majority voting with asymmetric weights to examine the endogenous choice of γ by the legislature of the central government. In the model, first, a representative for the legislature of the central government is selected through majority voting in each region. In the legislature, a representative from a more populous region carries greater weight in the legislative process.[23] The value of γ is decided using majority voting by the representatives. That is, region A's representative has more votes than region B's and is decisive in the choice of γ if and only if $n_A > n_B$. After the value of γ is determined, representatives for an interregional negotiation are selected and the negotiation is conducted, which is the same as the basic model. Without loss of generality, we assume that $n_A > n_B$, which implies that the decision-maker of γ is region A's representative.[24]

Our discussion is based on the numerical example presented in Example 1. In the extended model, we can show that the median resident of region A is decisive in the choice of γ in the central legislature.[25] Using numerical analyses based on Example 1, we examine the choice of γ by region A's median resident. We show how γ is related to n_A and a_M, taking $(n_B, b_M, \beta) = (1, 0.5, 0.5)$ as fixed. Because the second-order condition for region A's median resident holds if $1/\sqrt{3} \approx 0.577 < \gamma \leq 1$ (see (11)), we derive the optimal γ for region A's median resident, constrained on the interval $\left(1/\sqrt{3}, 1\right]$. Table 1 shows the relation between n_A, a_M, $\gamma[a_M]$(γ chosen by a_M), and γ^* (the Lindahl price in Theorem 1).[26]

[Insert Table 1 here]

[21]See Result 2 in the online appendix.

[22]Note that $u(x^{nb}, a_R) - \bar{u}^A$ and $v(x^{nb}, a_R) - \bar{v}^B$ are independent of β.

[23]An example of the distribution of votes according to the population is the European Council. Until November 1, 2014, for a decision subject to a qualified majority, the number of votes in a country was distributed according to its population; a decision passed with at least 260 out of 352 votes. Since then, the "double majority rule" has been adopted for a qualified majority rule, under which a decision needs approval by the representatives of those countries whose populations are at least 65 % of the total EU population. Hence, under this new system, essentially, a country with a large population is assigned a larger weight than that assigned to a country with a small population; see https://eur-lex.europa.eu/summary/glossary/weighting_votes_council.html.

[24]In their study on strategic delegation, Besley and Coate (2003) establish a model based on the "minimum winning coalition view" for the central legislature. This model yields the same result as ours if the more populous region is assumed to form the winning coalition. Lülfesmann (2002) provides a model of central legislature in which a more populous region is decisive in the decision of the legislature.

[25]See the detailed analysis in Section D in the online appendix.

[26]These analyses are conducted using Mathematica; the code is available upon request.

Table 1 shows that region A's median resident does not choose the Lindahl price as the cost-matching rate: $\gamma[a_M] < \gamma^*$. This is natural, because γ represents the cost-burden of the project for region A. Hence, region A has an incentive to decrease γ. However, we also find that $\gamma[a_M]$ is larger than $1/\sqrt{3}$ in every case, suggesting that a γ that is "too low" does not necessarily benefit region A's median resident.

Finally, note that who is decisive in the choice of γ depends on the model we adopt. In the model presented here, the median resident of a more populous region plays a decisive role in deciding γ. In other models, other residents may be decisive in the decision. However, our purpose is to examine the existence of the optimal cost-matching rate, not to show how γ depends on the choice of the model.

4.3 Population mobility

We briefly discuss the stability of the population distributions when the population is mobile. To do so, we consider the following model. Each region i ($i = A, B$) is populated with n_i mobile residents. The total population of the economy is denoted by $N = n_A + n_B$. Each individual has an attachment to one of the regions, which restricts their mobility across the regions. Following studies such as Mansoorian and Myers (1993), Silva and Yamaguchi (2010), and Boadway et al. (2013), each individual is characterized by a parameter n, which indicates an attachment to region A. The parameter n is assumed to be uniformly distributed over an interval $[0, N]$, such that $N > 0$. In contrast to previous studies, we assume this attachment value correlates to the benefit from the public project reaped by an individual. That is, as $n \in [0, N]$ increases, the individual with n becomes more strongly attached to region A. Because region A is the region undertaking the project, the resident in region A enjoys a greater benefit than that in region B, owing to the spillover of the project. Thus, the individual strongly attached to region A is the person receiving a high benefit from the project.

The timing of the game is as follows: In Stage 0, the central government sets the cost-matching rate of region A $\gamma \in [0, 1]$. After seeing this rate, in Stage 1, each individual chooses a region in which to reside. After the population distribution is determined, in Stage 2, one of the residents is chosen as the regional representative through majority voting in each region, and then the representatives negotiate. Stage 2 is the same as Stage 2 of the basic model.

As in the basic model, let x be a project level and T be the per-capita transfer that region B's residents make to region A. The individual with $n \in [0, N]$ receives the payoff

$$U(x, T; n) = n\mu(x) + \frac{I_A}{n_A} - \frac{\gamma}{n_A}c(x) + \frac{n_B}{n_A}T + tn$$

if he resides in region A, and receives the payoff

$$V(x, T; n) = n\mu(x) + \frac{I_B}{n_B} - \frac{1-\gamma}{n_B}c(x) - T + t(N - n)$$

if he resides in region B, where $t > 0$ is a parameter of attachment intensity. Hence, the term tn represents the attachment benefit of residing in region A, and the term $t(N - n)$ represents the attachment benefit of residing in region B. The benefit from the project consists of n and a concave function $\mu(x)$. The attachment benefits appear irrespective of whether the interregional negotiation succeeds or fails, indicating that the benefits do not affect the negotiation outcome. Therefore, the analysis of Stage 3 of this extended model is the same as that of Stage 2 of the basic model.

The condition of the migration equilibrium when it is interior is $U(x, T; n_m) = V(x, T; n_m)$, providing for the individual with the attachment n_m being indifferent between residing in regions A and B. Furthermore, Proposition 4 shows the stability of the population distribution generated by the migration equilibrium.

Proposition 4 *Suppose $n_m \in [0, N]$ satisfies the migration equilibrium condition $U(x^{nb}, T^{nb}; n_m) = V(x^{nb}, T^{nb}; n_m)$. Then, $U(x^{nb}, T^{nb}; n) > V(x^{nb}, T^{nb}; n)$, for all $n \in (n_m, N]$, and $U(x^{nb}, T^{nb}; n) <$*

$V(x^{nb}, T^{nb}; n)$, for all $n \in [0, n_m)$. Therefore, no individual is made better off by changing the residential choice.

The migration equilibrium determines the population distribution, such that $\mathcal{A} = [n_m, N]$ and $\mathcal{B} = [0, n_m]$. Proposition 4 states that the population distribution is stable in that no individuals are made better off by unilaterally changing their living regions. Our basic model is general enough to treat the population distribution at the migration equilibrium because no conditions are imposed on the relative position between $\mathcal{A}$ and $\mathcal{B}$ in the basic model.

Finally, we examine the kinds of population distributions supported at a migration equilibrium. This is discussed based on Example 1, because it is very difficult to characterize the equilibrium population distributions under our setting using general benefit and cost functions. We focus on the relation between the cost-matching rate γ and the equilibrium population distribution. We use the parameters of Table 1, with $n_B = 1, b_M = 0.5, \beta = 0.5, I_A = I_B = 10$, and $t = 1$. Here, γ^S in Table 2 is the cost-matching rate that attains the equilibrium condition $U(x^{nb}, T^{nb}; n_m) = V(x^{nb}, T^{nb}; n_m)$.[27] Table 2 shows the relation between n_A, a_M, and γ^S.

[Insert Table 2 here]

Table 2 shows an interesting tendency that, depending on the value γ, various population distributions may be stable. In particular, unlike in previous studies, regions A and B play asymmetric roles, where region A undertakes the project, but region B does not. Even under the asymmetric role, the number of residents may be the same, as shown in Case (1). However, different numbers of residents in both regions are also stable for some γs, as shown in Cases (2)–(7).

5 Conclusion

We have examined whether a cost-matching grant by a central government restores the efficiency of an interregional negotiation lost through the strategic delegation of representatives. We show that a grant achieves the efficient negotiation outcome if and only if (i) the cost-matching rate is the *Lindahl price*, and (ii) the manipulability of the negotiation breakdown outcome is sufficiently weak. The introduction of a grant generates a new kind of manipulation of negotiation breakdown outcomes. We find that the strength of the new manipulability is linked to whether the second-order condition for the median resident in the project region (region A) holds. Our result suggests a positive aspect to achieving an efficient project in that the central government obtains the optimal cost-matching rate if the manipulability is relatively weak. In addition, we show that the possibility of a re-election after a negotiation breaks down helps the cost-matching grant yield an efficient project. This re-election adjusts the breakdown outcome such that an optimal cost-matching rate that achieves an efficient project always exists. To the best of our knowledge, few studies on strategic delegation problems have considered whether a government policy alleviates the strategic delegation problem. Thus, our contribution is to clarify the conditions under which a central government may or may not obtain the optimal grant that solves the strategic delegation problem.

Several problems remain that need to be addressed in future work. It is important to consider how the central government implements the optimal cost-matching grant. From our results, the optimal grant depends on the regions' benefit information, which is usually private information. Hence, it might be necessary to extract such information by applying methods of mechanism design theory. Furthermore, an extension to our setting that includes multiple regions that benefit from public projects would also be interesting. The models of previous studies (e.g., Ray and Vohra, 1997, 2001; Dixit and Olson, 2000; Matsushima and Shinohara, 2019) might be useful to this extension.

[27]The value of γ^S is calculated using Mathematica; the code is available upon request.

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	n_A	a_M	$\gamma[a_M]$	γ^*
Case (1)	1	1.5	0.586363	0.75
Case (2)	1.5	1.75	0.595239	0.84
Case (3)	2	2	0.598785	0.888889
Case (4)	2.5	2.25	0.600621	0.918367
Case (5)	3	2.5	0.601721	0.9375
Case (6)	3.5	2.75	0.602418	0.950617
Case (7)	4	3	0.602904	0.96

Table 1: The constrained optimal cost-matching rate for region A's median resident

	n_A	a_M	γ^S
Case (1)	1	1.5	0.581179
Case (2)	1.5	1.75	0.600014
Case (3)	2	2	0.638465
Case (4)	2.5	2.25	0.677132
Case (5)	3	2.5	0.712904
Case (6)	3.5	2.75	0.744515
Case (7)	4	3	0.771707

Table 2: The migration equilibrium and the cost-matching rate

Appendix

Single-crossing properties of preferences

We show that residents' preferences, represented by $U(x^{nb}, T^{nb}; a)$ and $V(x^{nb}, T^{nb}; b)$ in (6) and (7), respectively, in Section 3.2, satisfy the single-crossing property of Gans and Smart (1996).

Let $a_R, a'_R, a', a \in \mathcal{A}$, such that $a_R > a'_R$ and $a' > a$. Let $b_R \in \mathcal{B}$. We show that

$$\begin{aligned}
&\text{if } U(x^{nb}(a_R, b_R), T^{nb}(a_R, b_R); a) \geq U(x^{nb}(a'_R, b_R), T^{nb}(a'_R, b_R); a),\\
&\text{then } U(x^{nb}(a_R, b_R), T^{nb}(a_R, b_R); a') > U(x^{nb}(a'_R, b_R), T^{nb}(a'_R, b_R); a').
\end{aligned}$$

By the hypothesis,

$$\begin{aligned}
u(x^{nb}(a_R, b_R), a) - u(x^{nb}(a'_R, b_R), a) \geq &\frac{\gamma}{n_A}\left(c(x^{nb}(a_R, b_R), a) - c(x^{nb}(a'_R, b_R), a)\right)\\
&-\frac{n_B}{n_A}\left(T^{nb}(a_R, b_R) - T^{nb}(a'_R, b_R)\right).
\end{aligned} \tag{17}$$

By $a_R > a'_R$, $x^{nb}(a_R, b_R) > x^{nb}(a'_R, b_R)$, $a' > a$, and $u_{xa} > 0$,

$$u(x^{nb}(a_R, b_R), a) - u(x^{nb}(a'_R, b_R), a) < u(x^{nb}(a_R, b_R), a') - u(x^{nb}(a'_R, b_R), a').$$

In conclusion,

$$\begin{aligned}
u(x^{nb}(a_R, b_R), a') - u(x^{nb}(a'_R, b_R), a') > &\frac{\gamma}{n_A}\left(c(x^{nb}(a_R, b_R), a) - c(x^{nb}(a'_R, b_R), a)\right)\\
&-\frac{n_B}{n_A}\left(T^{nb}(a_R, b_R) - T^{nb}(a'_R, b_R)\right),
\end{aligned}$$

which implies $U(x^{nb}(a_R, b_R), T^{nb}(a_R, b_R); a') > U(x^{nb}(a'_R, b_R), T^{nb}(a'_R, b_R); a')$.

Similarly, we can show that for all $a_R \in \mathcal{A}$ and all $b_R, b'_R, b', b \in \mathcal{B}$, such that $b_R > b'_R$ and $b' > b$, $V(x^{nb}(a_R, b_R), T^{nb}(a_R, b_R); b) \geq V(x^{nb}(a_R, b'_R), T^{nb}(a_R, b'_R); b)$ implies $V(x^{nb}(a_R, b_R), T^{nb}(a_R, b_R); b') > V(x^{nb}(a_R, b'_R), T^{nb}(a_R, b'_R); b')$.

Similarly, the preferences represented by $U(x^{nb}, T^{nb}; a)$ and $V(x^{nb}, T^{nb}; b)$ in Section 3.3 can be shown to be single-crossing. Note that the right-hand side of (17) is common to all residents in region A in any model.

Proofs

Proof of Lemma 1

Differentiating the Nash product with respect to x and T, we obtain from the first-order condition that

$$\frac{\beta\left(u_x(x^{nb}, a_R) - \frac{\gamma}{n_A}c'(x^{nb})\right)}{u(x^{nb}, a_R) + \frac{I_A}{n_A} - \frac{\gamma}{n_A}c(x^{nb}) + \frac{n_B}{n_A}T^{nb} - U\left(x^A, 0; a_R\right)} + \frac{(1-\beta)\left(v_x(x^{nb}, b_R) - \frac{1-\gamma}{n_B}c'(x^{nb})\right)}{v(x^{nb}, b_R) + \frac{I_B}{n_B} - \frac{1-\gamma}{n_B}c(x^{nb}) - T^{nb} - V\left(x^A, 0; b_R\right)} = 0$$

and

$$\frac{\beta\left(\frac{n_B}{n_A}\right)}{u(x^{nb}, a_R) + \frac{I_A}{n_A} - \frac{\gamma}{n_A}c(x^{nb}) + \frac{n_B}{n_A}T^{nb} - U\left(x^A, 0; a_R\right)} = \frac{1-\beta}{v(x^{nb}, b_R) + \frac{I_B}{n_B} - \frac{1-\gamma}{n_B}c(x^{nb}) - T^{nb} - V\left(x^A, 0; b_R\right)}.$$

Combining these conditions yields (4) and (5).

Proof of Lemma 2

First, note that the differential coefficient of x^A with respect to a_R and that of x^{nb} with respect to a_R and b_R are positive. As (3) shows, $\partial x^A(a_R,\gamma)/\partial a_R$ is positive. Differentiating (4) with respect to a_R and b_R yields

$$\begin{aligned} \frac{\partial x^{nb}(a_R,b_R)}{\partial a_R} &= -\frac{n_A u_{xa}(x^{nb},a_R)}{n_A u_{xx}(x^{nb},a_R)+n_B v_{xx}(x^{nb},b_R)-c''(x^{nb})} > 0 \\ \text{and}\quad \frac{\partial x^{nb}(a_R,b_R)}{\partial b_R} &= -\frac{n_B v_{xb}(x^{nb},b_R)}{n_A u_{xx}(x^{nb},a_R)+n_B v_{xx}(x^{nb},b_R)-c''(x^{nb})} > 0. \end{aligned} \tag{18}$$

Differentiating $U\left(x^{nb},T^{nb};a_M\right)$ with respect to a_R yields

$$\begin{aligned} \frac{\partial\, U(x^{nb},T^{nb};a_M)}{\partial\, a_R} &= \frac{\partial x^{nb}}{\partial a_R}\left[u_x(x^{nb},a_M)-u_x(x^{nb},a_R)+\frac{\beta}{n_A}\left(n_A u_x(x^{nb},a_R)+n_B v_x(x^{nb},b_R)-c'(x^{nb})\right)\right] \\ &+\frac{\partial x^A}{\partial a_R}\left[u_x(x^A,a_R)-\frac{\gamma}{n_A}c'(x^A)-\frac{\beta}{n_A}(n_A u_x(x^A,a_R)+n_B v_x(x^A,b_R)-c'(x^A))\right] \\ &-(1-\beta)\left[u_a(x^{nb},a_R)-u_a(x^A,a_R)\right] \\ &= \frac{\partial x^{nb}}{\partial a_R}\left[u_x(x^{nb},a_M)-u_x(x^{nb},a_R)\right] \\ &-\left(\frac{\beta}{n_A}\right)\frac{\partial x^A}{\partial a_R}\left[n_A u_x(x^A,a_R)+n_B v_x(x^A,b_R)-c'(x^A)\right] \\ &-(1-\beta)\left[u_a(x^{nb},a_R)-u_a(x^A,a_R)\right]. \end{aligned}$$

The last equality follows from the first-order conditions (2) and (4).

Similarly, from

$$\begin{aligned} \frac{\partial\, V(x^{nb},T^{nb};b_M)}{\partial\, b_R} &= \frac{\partial x^{nb}}{\partial b_R}\left[v_x(x^{nb},b_M)-v_x(x^{nb},b_R)\right]-v_b(x^{nb},b_R)+v_b(x^A,b_R) \\ &+\left(\frac{1-\beta}{n_B}\right)\frac{\partial x^{nb}}{\partial b_R}\left[n_A u_x(x^{nb},a_R)+n_B v_x(x^{nb},b_R)-c'(x^{nb})\right] \\ &+(1-\beta)\left[v_b(x^{nb},b_R)-v_b(x^A,b_R)\right], \end{aligned}$$

we obtain (10).

Proof of Proposition 1

From (2) and (4), it follows that for any pair $(a_R,b_R)\in\mathcal{A}\times\mathcal{B}$, $x^A(a_R,1)<x^{nb}(a_R,b_R)$. By $x^A(a_R,1)<x^{nb}(a_R,b_R)$,

$$n_A u_x(x^A,a_R)+n_B v_x(x^A,b_R)-c'(x^A)>0 \text{ and } u_a(x^{nb},a_R)-u_a(x^A,a_R)>0.$$

By these conditions, the second and third lines in (9) are negative. Hence, in order for the first-order condition to be met, it follows that $u_x(x^{nb},a_M)-u_x(x^{nb},a_R^*)>0$, which implies $a_M>a_R^*$. Similarly, from (10), we find that $b_M\geq b_R^*$ with equality if $\beta=0$. ■

Proof of Theorem 1

We establish several steps to show this theorem. First, we show Lemma3.

Proof of Lemma 3. Because $n_A u_x(x^{nb}(a_M,b_M),a_M)+n_B v_x(x^{nb}(a_M,b_M),b_M)=c'(x^{nb}(a_M,b_M))$, we have

$$u_x(x^{nb}(a_M,b_M),a_M)=\left(\frac{n_A u_x(x^{nb}(a_M,b_M),a_M)}{n_A u_x(x^{nb}(a_M,b_M),a_M)+n_B v_x(x^{nb}(a_M,b_M),b_M)}\right)\frac{1}{n_A}c'(x^{nb}(a_M,b_M)).$$

If $\gamma \geq \gamma^*$, then $u_x(x^{nb}(a_M, b_M), a_M) \leq (\gamma/n_A)c'(x^{nb}(a_M, b_M))$. Because $u_x(x^A(a_M, \gamma), a_M) = (\gamma/n_A)c'(x^A(a_M, \gamma))$ and u_x is nonincreasing and c' is increasing in x, we find that $x^A(a_M, \gamma) \leq x^{nb}(a_M, b_M)$. Similarly, we find that if $\gamma < \gamma^*$, then $x^A(a_M, \gamma) > x^{nb}(a_M, b_M)$. ■

Claim 1 provides the condition under which the self-representation is a solution of the first-order conditions.

Claim 1 $(a_R, b_R) = (a_M, b_M)$ is a solution of the first-order conditions (8) if and only if $\gamma = \gamma^*$.

Proof of Claim 1. (*If-part*) If $\gamma = \gamma^*$, then $x^A(a_M, \gamma^*) = x^{nb}(a_M, b_M)$, by Lemma 3. Then,

$$u_a(x^{nb}(a_M, b_M), a_M) = u_a(x^A(a_M, \gamma^*), a_M), \; v_b(x^{nb}(a_M, b_M), b_M) = v_b(x^A(a_M, \gamma^*), b_M),$$
$$\text{and } u_x(x^A(a_M, \gamma^*), a_M) + v_x(x^A(a_M, \gamma^*), b_M) = c'(x^A(a_M, \gamma^*)).$$

If $\gamma = \gamma^*$ and $(a_R, b_R) = (a_M, b_M)$, then by (9),

$$\begin{aligned}
\frac{\partial U(x^{nb}, T^{nb}; a_M)}{\partial a_R} &= \frac{\partial x^{nb}}{\partial a_R}\left[u_x(x^{nb}(a_M, b_M), a_M) - u_x(x^{nb}(a_M, b_M), a_M)\right] \\
&\quad - \left(\frac{\beta}{n_A}\right)\frac{\partial x^A}{\partial a_R}\left[n_A u_x(x^A(a_M, \gamma^*), a_M) + n_B v_x(x^A(a_M, \gamma^*), b_M) - c'(x^A(a_M, \gamma^*))\right] \\
&\quad - (1-\beta)\left[u_a(x^{nb}(a_M, b_M), a_M) - u_a(x^A(a_M, \gamma^*), a_M)\right] = 0,
\end{aligned}$$

and by (10),

$$\begin{aligned}
\frac{\partial V(x^{nb}, T^{nb}; b_M)}{\partial b_R} &= \frac{\partial x^{nb}}{\partial b_R}\left[v_x(x^{nb}(a_M, b_M), b_M) - v_x(x^{nb}(a_M, b_M), b_M)\right] \\
&\quad - \beta\left[v_b(x^{nb}(a_M, b_M), b_M) - v_b(x^A(a_M, \gamma), b_M)\right] = 0.
\end{aligned}$$

Thus, $(a_R, b_R) = (a_M, b_M)$ satisfies (8).

(*Only-if-part*) Suppose $(a_R, b_R) = (a_M, b_M)$, but that $\gamma \neq \gamma^*$. Without loss of generality, suppose $\gamma > \gamma^*$. Denote $x^{nb} = x^{nb}(a_M, b_M)$ and $x^A = x^A(a_M, \gamma)$, for simplicity. Then, $x^{nb} > x^A$, which implies $n_A u_x(x^A, a_M) + n_B v_x(x^A, b_M) - c'(x^A) > 0$. Because $u_{ax} > 0$ and $v_{bx} > 0$, we have $u_a(x^{nb}, a_M) - u_a(x^A, a_M) > 0$ and $v_b(x^{nb}, b_M) - v_b(x^A, b_M) > 0$. Then, from (9) and (10),

$$\begin{aligned}
\left.\frac{\partial U(x^{nb}, T^{nb}, a_M)}{\partial a_R}\right|_{(a_R, b_R) = (a_M, b_M)} &= -\left(\frac{\beta}{n_A}\right)\frac{\partial x^A}{\partial a_R}\left[n_A u_x(x^A, a_M) + n_B v_x(x^A, b_M) - c'(x^A))\right] \\
&\quad - (1-\beta)\left[u_a(x^{nb}, a_M) - u_a(x^A, a_M)\right] < 0
\end{aligned}$$

and

$$\left.\frac{\partial V(x^{nb}, T^{nb}; b_M)}{\partial b_R}\right|_{(a_R, b_R) = (a_M, b_M)} = -\beta\left(v_b(x^{nb}, b_M) - v_b(x^A, b_M)\right) \leq 0.$$

Hence, $(a_R, b_R) = (a_M, b_M)$ does not satisfy (8), which is a contradiction. ■

In Claim 2, we show that the second-order condition for region B's median resident is always satisfied at $\gamma = \gamma^*$ and $(a_R, b_R) = (a_M, b_M)$.

Claim 2 It follows that

$$\left.\frac{\partial^2 V(x^{nb}, T^{nb}; b_M)}{\partial b_R^2}\right|_{(a_R, b_R) = (a_M, b_M), \gamma = \gamma^*} < 0.$$

Proof of Claim 2. Differentiating (10) with respect to b_R yields

$$\begin{aligned}\frac{\partial^2 V(x^{nb},T^{nb};b_M)}{\partial b_R^2} &= \frac{\partial^2 x^{nb}}{\partial b_R^2}\left(v_x(x^{nb},b_M)-v_x(x^{nb},b_R)\right)\\&\quad+\left(\frac{\partial x^{nb}}{\partial b_R}\right)^2\left(v_{xx}(x^{nb},b_M)-v_{xx}(x^{nb},b_R)\right)-v_{bx}(x^{nb},b_R)\frac{\partial x^{nb}}{\partial b_R}\\&\quad-\beta\left(v_{bb}(x^{nb},b_R)-v_{bb}(x^A,b_R)+v_{bx}(x^{nb},b_R)\frac{\partial x^{nb}}{\partial b_R}\right).\end{aligned} \tag{19}$$

At $\gamma=\gamma^*$ and $(a_R,b_R)=(a_M,b_M)$, $x^{nb}(a_M,b_M)=x^A(a_M,\gamma^*)$, by Lemma 3. Thus, by (19), we have

$$\frac{\partial^2 V(x^{nb},T^{nb};b_M)}{\partial b_R^2} = -v_{bx}(x^{nb},b_M)\frac{\partial x^{nb}}{\partial b_R}-\beta\left(v_{bx}(x^{nb},b_M)\frac{\partial x^{nb}}{\partial b_R}\right),$$

which is negative because $\partial x^{nb}/\partial b_R>0$ and $v_{bx}(x^{nb},b_M)>0$. ■

In Claim 3, we provide a necessary and sufficient condition for the second-order condition of region A's median resident to be satisfied.

Claim 3 There exists a positive real $\bar{X}$, such that

$$\left.\frac{\partial^2 U(x^{nb},T^{nb};a_M)}{\partial a_R^2}\right|_{(a_R,b_R)=(a_M,b_M),\gamma=\gamma^*}<0 \text{ if and only if } \left.\frac{\partial x^A(a_R,\gamma)}{\partial a_R}\right|_{a_R=a_M,\gamma=\gamma^*}<\bar{X}.$$

Proof of Claim 3. Differentiating (9) with respect to a_R yields

$$\begin{aligned}\frac{\partial^2 U(x^{nb},T^{nb};a_M)}{\partial a_R^2} &= \frac{\partial^2 x^{nb}}{\partial a_R^2}\left(u_x(x^{nb},a_M)-u_x(x^{nb},a_R)\right)\\&\quad+\left(\frac{\partial x^{nb}}{\partial a_R}\right)^2\left(u_{xx}(x^{nb},a_M)-u_{xx}(x^{nb},a_R)\right)\\&\quad-u_{xa}(x^{nb},a_R)\frac{\partial x^{nb}}{\partial a_R}\\&\quad-\left(\frac{\beta}{n_A}\right)\frac{\partial^2 x^A}{\partial a_R^2}\left(n_Au_x(x^A,a_R)+n_Bv_x(x^A,b_R)-c'(x^A)\right)\\&\quad-\left(\frac{\beta}{n_A}\right)\left(\frac{\partial x^A}{\partial a_R}\right)^2\left(n_Au_{xx}(x^A,a_R)+n_Bv_{xx}(x^A,b_R)-c''(x^A)\right)\\&\quad-n_Au_{xa}(x^A,a_R)\left(\frac{\beta}{n_A}\right)\frac{\partial x^A}{\partial a_R}\\&\quad-(1-\beta)\left(u_{aa}(x^{nb},a_R)+u_{ax}(x^{nb},a_R)\frac{\partial x^{nb}}{\partial a_R}-u_{aa}(x^A,a_R)-u_{ax}(x^A,a_R)\frac{\partial x^A}{\partial a_R}\right),\end{aligned} \tag{20}$$

where $x^{nb}=x^{nb}(a_R,b_R)$ and $x^A=x^A(a_R,\gamma)$. Substituting $(a_R,b_R)=(a_M,b_M)$ and $\gamma=\gamma^*$ into (20) yields

$$\begin{aligned}\frac{\partial^2 U(x^{nb},T^{nb};a_M)}{\partial a_R^2} &= -u_{xa}(x^{nb},a_M)\frac{\partial x^{nb}}{\partial a_R}\\&\quad-\left(\frac{\beta}{n_A}\right)\left(\frac{\partial x^A}{\partial a_R}\right)^2\left(n_Au_{xx}(x^A,a_M)+n_Bv_{xx}(x^A,b_M)-c''(x^A)\right)\\&\quad-u_{xa}(x^A,a_M)\beta\frac{\partial x^A}{\partial a_R}-(1-\beta)\left(u_{ax}(x^{nb},a_M)\frac{\partial x^{nb}}{\partial a_R}-u_{ax}(x^A,a_M)\frac{\partial x^A}{\partial a_R}\right),\end{aligned}$$

because on the right-hand side of (20), the first and second lines are zero, the fourth line is zero by $x^A(a_M, \gamma^*) = x^{nb}(a_M, b_M)$, and $u_{aa}(x^A, a_M) = u_{aa}(x^{nb}, a_M) = 0$. Rearranging (20) yields

$$\begin{aligned}\frac{\partial^2 U(x^{nb}, T^{nb}; a_M)}{\partial a_R^2} &= -\left(\frac{\beta}{n_A}\right)\left(n_A u_{xx}(x^A, a_M) + n_B v_{xx}(x^A, b_M) - c''(x^A)\right)\left(\frac{\partial x^A}{\partial a_R}\right)^2 \\ &\quad + (1-2\beta) u_{xa}(x^A, a_M)\frac{\partial x^A}{\partial a_R} + (\beta - 2) u_{xa}(x^{nb}, a_M)\frac{\partial x^{nb}}{\partial a_R}.\end{aligned} \tag{21}$$

The right-hand side of (21) can be viewed as a quadratic function of $\partial x^A/\partial a_R$. Note that in (21), the coefficient of $(\partial x^A/\partial a_R)^2$ is positive and the final term is negative, implying there is a positive real $\bar{X}$, such that $\partial^2 U(x^{nb}, T^{nb}; a_M)/\partial a_R^2 < 0$ if and only if $\partial x^A/\partial a_R < \bar{X}$. ■

Claim 3 indicates that at $(a_R, b_R) = (a_M, b_M)$ and $\gamma = \gamma^*$, the second-order condition for region A's median resident is satisfied if and only if the manipulability of the breakdown level of the project x^A through the choice of a_R is sufficiently weak.

Proof of Proposition 2

Suppose that $u(x, a) = ax$, $v(x, b) = bx$, and $c(x) = x^\alpha/\alpha$ $(\alpha \geq 2)$. Then, from (19) and (20), the second-order derivatives of $U(x^{nb}, T^{nb}; a_M)$ and $V(x^{nb}, T^{nb}; a_M)$ are calculated as follows:

$$\begin{aligned}\frac{\partial^2 U(x^{nb}, T^{nb}; a_M)}{\partial a_R^2} &= \frac{\partial^2 x^{nb}}{\partial a_R^2}(a_M - a_R) - \frac{\partial x^{nb}}{\partial a_R} - \left(\frac{\beta}{n_A}\right)\frac{\partial^2 x^A}{\partial a_R^2}\left(n_A a_R + n_B b_R - c'(x^A)\right) \\ &\quad + \left(\frac{\beta}{n_A}\right)\left(\frac{\partial x^A}{\partial a_R}\right)^2 c''(x^A) - \beta\frac{\partial x^A}{\partial a_R} - (1-\beta)\left(\frac{\partial x^{nb}}{\partial a_R} - \frac{\partial x^A}{\partial a_R}\right)\end{aligned} \tag{22}$$

$$\text{and } \frac{\partial^2 V(x^{nb}, T^{nb}; b_M)}{\partial b_R^2} = \frac{\partial^2 x^{nb}}{\partial b_R^2}(b_M - b_R) - (1+\beta)\frac{\partial x^{nb}}{\partial b_R}. \tag{23}$$

Because $x^{nb}(a_R, b_R) = (n_A a_R + n_B b_R)^{\frac{1}{\alpha-1}}$ and $x^A(a_R, \gamma) = (n_A a_R/\gamma)^{\frac{1}{\alpha-1}}$, we have

$$\begin{aligned}&\frac{\partial x^{nb}(a_R, b_R)}{\partial a_R} = \frac{n_A(n_A a_R + n_B b_R)^{\frac{2-\alpha}{\alpha-1}}}{\alpha - 1}, \quad \frac{\partial x^{nb}(a_R, b_R)}{\partial b_R} = \frac{n_B(n_A a_R + n_B b_R)^{\frac{2-\alpha}{\alpha-1}}}{\alpha - 1}, \\ &\frac{\partial^2 x^{nb}(a_R, b_R)}{\partial a_R^2} = \frac{n_A^2(2-\alpha)(n_A a_R + n_B b_R)^{\frac{3-2\alpha}{\alpha-1}}}{(\alpha-1)^2}, \quad \frac{\partial^2 x^{nb}(a_R, b_R)}{\partial b_R^2} = \frac{n_B^2(2-\alpha)(n_A a_R + n_B b_R)^{\frac{3-2\alpha}{\alpha-1}}}{(\alpha-1)^2}, \\ &\frac{\partial x^A(a_R, \gamma)}{\partial a_R} = \left(\frac{n_A}{\gamma}\right)^{\frac{1}{\alpha-1}}\left(\frac{(a_R)^{\frac{2-\alpha}{\alpha-1}}}{\alpha - 1}\right), \text{ and } \frac{\partial^2 x^A(a_R, \gamma)}{\partial a_R^2} = \left(\frac{n_A}{\gamma}\right)^{\frac{1}{\alpha-1}}\left(\frac{(2-\alpha)(a_R)^{\frac{3-2\alpha}{\alpha-1}}}{(\alpha-1)^2}\right).\end{aligned} \tag{24}$$

Substituting the values of (24) into (23) yields

$$\frac{\partial^2 V(x^{nb}, T^{nb}; b_M)}{\partial b_R^2} = \frac{n_B(n_A a_R + n_B b_R)^{\frac{3-2\alpha}{\alpha-1}}\left(n_B b_M(2-\alpha) - n_A a_R(\alpha-1)(\beta+1) + n_B b_R((1-\alpha)\beta - 1)\right)}{(\alpha-1)^2},$$

which is negative, by $\alpha \geq 2$ and $0 \leq \beta \leq 1$.

Finally, we show that if $\partial x^A(a_R, \gamma)/\partial a_R$ is sufficiently small, then $\partial^2 U(x^{nb}, T^{nb}; a_M)/\partial a_R^2 < 0$. As a first step, we consider an "imaginary" situation in which $\partial x^A(a_R, \gamma)/\partial a_R = 0$ holds.

Claim 4 If $\partial x^A(a_R, \gamma)/\partial a_R = 0$, then $\partial^2 U(x^{nb}, T^{nb}; a_M)/\partial a_R^2 < 0$.

Proof of Claim 4. First, because $\partial x^A(a_R,\gamma)/\partial a_R = 0$, from (22), we obtain that

$$\frac{\partial^2 U(x^{nb},T^{nb};a_M)}{\partial a_R^2} = \frac{\partial^2 x^{nb}}{\partial a_R^2}(a_M - a_R) - (2-\beta)\frac{\partial x^{nb}}{\partial a_R}.$$

Then, substituting the values of (24) into the previous formula yields

$$\frac{\partial^2 U(x^{nb},T^{nb};a_M)}{\partial a_R^2} = \frac{n_A(n_A a_R + n_B b_R)^{\frac{3-2\alpha}{\alpha-1}}\,(n_A a_M(2-\alpha) + n_A a_R(\alpha(\beta-1)-\beta) + n_B b_R(\alpha-1)(\beta-2))}{(\alpha-1)^2},$$

which is negative, by $\alpha \geq 2$ and $0 \leq \beta \leq 1$. ■

Here, $\partial^2 U(x^{nb},T^{nb};a_M)/\partial a_R^2$ in (22) can be viewed as a quadratic function of $\partial x^A/\partial a_R$. Note that the coefficient of $\left(\partial x^A/\partial a_R\right)^2$ is positive, because $\left(\beta c''(x^A)/n_A\right) > 0$. In addition, by Claim 4, $\partial^2 U(x^{nb},T^{nb};a_M)/\partial a_R^2 < 0$ if the value of $\partial x^A/\partial a_R$ is zero. Thus, for each $(a_R,b_R) \in \mathcal{A}\times\mathcal{B}$, there exists a positive threshold value $\bar{X}(a_R,b_R)$, such that $\partial^2 U(x^{nb},T^{nb};a_M)/\partial a_R^2 < 0$ if and only if

$$\frac{\partial x^A(a_R,\gamma)}{\partial a_R} < \bar{X}(a_R,b_R).$$

The value $\bar{X}(a_R,b_R)$ is continuous in (a_R,b_R) because $\partial^2 U(x^{nb},T^{nb};a_M)/\partial a_R^2$ is continuous with respect to (a_R,b_R). Furthermore, $\mathcal{A}\times\mathcal{B} = [\underline{a},\bar{a}]\times[\underline{b},\bar{b}]$ is a compact set. From these facts, and applying the Weierstrass extreme value theorem, we obtain the result that $\bar{X}(a_R,b_R)$ has a minimum value at some point in $\mathcal{A}\times\mathcal{B}$. Therefore, we find that for each $(a_R,b_R) \in \mathcal{A}\times\mathcal{B}$, $\partial^2 U(x^{nb},T^{nb};a_M)/\partial a_R^2 < 0$ if $\partial x^A/\partial a_R < \min_{(a_R,b_R)\in\mathcal{A}\times\mathcal{B}} \bar{X}(a_R,b_R)$. Therefore, $U(x^{nb},T^{nb};a_M)$ is concave in a_R when $\partial x^A/\partial a_R$ is sufficiently small.

Proof of Proposition 3

We analyze the game by backward induction.

First, we analyze Stage 3. Suppose $\tilde{a}_R \in \mathcal{A}$ is elected as the setter of the disagreement level of the project. Then, she chooses $x^A = x^A(\tilde{a}_R,\gamma)$, such that $u_x(x^A(\tilde{a}_R,\gamma),\tilde{a}_R) = \gamma c'(x^A(\tilde{a}_R,\gamma))/n_A$. Given this level of the project, each resident $a \in \mathcal{A}$ obtains a payoff of $U(x^A(\tilde{a}_R,\gamma),0;a) = u(x^A(\tilde{a}_R,\gamma),a) - \gamma c(x^A(\tilde{a}_R,\gamma))/n_A + I_A/n_A$. The optimal representative for $a \in \mathcal{A}$ in the election in Stage 3 is determined by the first-order condition,

$$\frac{\partial x^A}{\partial \tilde{a}_R}\left(u_x(x^A(\tilde{a}_R,\gamma),a) - \frac{\gamma}{n_A}c'(x^A(\tilde{a}_R,\gamma))\right) = 0.$$

The solution is given by $\tilde{a}_R = a$; that is, the optimal representative for each region A's resident is herself.[28] We can easily check that the preference represented by $U(x^A(\tilde{a}_R,\gamma),a)$ satisfies the single-crossing condition of Gans and Smart (1996). Hence, in this election stage, the decisive voter a_M chooses herself as the regional representative, and the project level achieved through the new election is $x^A(a_M,\gamma)$. Note that the breakdown level of the project is fixed at $\bar{x}^A \equiv x^A(a_M,\gamma)$ if the negotiation fails. Therefore, the choice of the negotiation representative in Stage 1 does not affect the disagreement level of the project; that is, $\partial\bar{x}^A/\partial a_R = 0$.

[28]The second derivative of $U(x^A(\tilde{a}_R,\gamma),0;a)$ with respect to $\tilde{a}_R$ is

$$\begin{aligned}\frac{\partial^2 U(x^A(\tilde{a}_R,\gamma),0;a)}{\partial \tilde{a}_R^2} &= \frac{\partial^2 x^A}{\partial \tilde{a}_R^2}\left(u_x(x^A(\tilde{a}_R,\gamma),a) - \frac{\gamma}{n_A}c'(x^A(\tilde{a}_R,\gamma))\right) \\ &+ \left(\frac{\partial x^A}{\partial \tilde{a}_R}\right)^2\left(u_{xx}(x^A(\tilde{a}_R,\gamma),a) - \frac{\gamma}{n_A}c''(x^A(\tilde{a}_R,\gamma))\right).\end{aligned}$$

The first term is equal to zero at $\tilde{a}_R = a$, from the first-order condition. The second term is negative, from the conditions of u_{xx} and c''. Hence, $\partial^2 U(x^A(\tilde{a}_R,\gamma),0;a)/\partial\tilde{a}_R^2|_{\tilde{a}_R=a} < 0$. Similarly to Proposition 2, we can prove that $U(x^A(\tilde{a}_R,\gamma),a)$ is concave in $\tilde{a}_R$ if $u(x,a) = ax$ and $c(x) = x^\alpha/\alpha$, such that $\alpha \geq 2$.

Finally, we analyze Stages 1 and 2. We can apply some parts of the proofs of Theorem 1 and Proposition 2. If we substitute zero in each $\partial x^A/\partial a_R$ in the proof of Claim 1, then we obtain that $(a_R^*, b_R^*) = (a_M, b_M)$ is a solution of (8) if and only if $\gamma = \gamma^*$. Substituting zero in $\partial x^A/\partial a_R$ of (21), we can prove that the second-order condition for region A's median resident holds. By directly applying Claim 2, we find that the second-order condition for region B's median resident is satisfied. In a similar way to the proof of Proposition 2, the payoff functions of the median residents are proven to be concave in their representatives when $u(x,a) = ax$, $v(x,b) = bx$, and $c(x) = x^\alpha/\alpha$, such that $\alpha \geq 2$. ■

Proof of Proposition 4

From $U(x^{nb}, T^{nb}; n_m) = V(x^{nb}, T^{nb}; n_m)$, we have

$$\frac{I_A}{n_A} - \frac{\gamma c(x^{nb})}{n_A} + \frac{n_B T^{nb}}{n_A} + tn_m = \frac{I_B}{n_B} - \frac{(1-\gamma)c(x^{nb})}{n_B} - T^{nb} + t(N - n_m).$$

If $n > n_m$, then we have

$$\begin{aligned}\frac{I_A}{n_A} - \frac{\gamma c(x^{nb})}{n_A} + \frac{n_B T^{nb}}{n_A} + tn &> \frac{I_A}{n_A} - \frac{\gamma c(x^{nb})}{n_A} + \frac{n_B T^{nb}}{n_A} + tn_m \\ &= \frac{I_B}{n_B} - \frac{(1-\gamma)c(x^{nb})}{n_B} - T^{nb} + t(N - n_m) \\ &> \frac{I_B}{n_B} - \frac{(1-\gamma)c(x^{nb})}{n_B} - T^{nb} + t(N - n).\end{aligned}$$

Adding $n\mu(x^{nb})$ to the left-hand and right-hand sides of the above conditions yields $U(x^{nb}, T^{nb}; n) > V(x^{nb}, T^{nb}; n)$. We can show the case of $n < n_m$ in a similar manner. ■

Online Appendix

A On the second-order condition for region A's median resident

We consider the case of $u(x,a) = ax$, $v(x,b) = bx$, and $c(x) = x^3/3$. We derive the second-order conditions for region A's median resident for $\mathcal{A}$ and $\mathcal{B}$ in Table 1.[1)] Note that $n_B = 1$, $b_M = 1/2$, and $\beta = 1/2$. The rightmost column shows the second-order condition for region A's median resident for each case.

	n_A	a_M	SOC for a_M
Case (1)	1	1.5	$0.458024 < \gamma \leq 1$
Case (2)	1.5	1.75	$0.420769 < \gamma \leq 1$
Case (3)	2	2	$0.404444 < \gamma \leq 1$
Case (4)	2.5	2.25	$0.395618 < \gamma \leq 1$
Case (5)	3	2.5	$0.390245 < \gamma \leq 1$
Case (6)	3.5	2.75	$0.38671 < \gamma \leq 1$
Case (7)	4	3	$0.384251 < \gamma \leq 1$

Table 3: The second-order condition for region A's median resident when $c(x) = x^3/3$.

Note that as region A's population increases, the interval of the condition expands. Hence, the relative position of $\mathcal{A}$ and $\mathcal{B}$ may have an effect on the second-order condition for region A's median resident. This is in contrast to the case of Example 1.

B The case in which region A's second-order condition is violated

Proof of Result 1. Claim A1 shows the best response function of region A's median resident.

Claim A1 (i) If $0 \leq \gamma \leq a_M/(1+a_M)$, then $a_R^*(b_R) = \bar{a}$, for all $b_R \in \mathcal{B}$. (ii) If $1/\sqrt{3} > \gamma > a_M/(1+a_M)$, then

$$a_R^*(b_R) = \begin{cases} \bar{a} & \text{if } 0 \leq b_R \leq \frac{a_M(1-\gamma)}{\gamma} \\ \underline{a} & \text{if } 1 \geq b_R \geq \frac{a_M(1-\gamma)}{\gamma} \end{cases}.$$

Proof of Claim A1. As we can see from (11), the payoff function of the median resident of region A $U(x^{nb}, T^{nb}; a_M)$ is a quadratic *convex* function with regard to a_R, and is minimized at

$$a_R = \widehat{a}_R \equiv \frac{2\gamma^2}{3\gamma^2-1}a_M - \frac{\gamma(1+\gamma)}{3\gamma^2-1}b_R.$$

Thus, the best response of region A's median resident must be a corner solution, and is either $\underline{a}$ or $\bar{a}$. Because $U(x^{nb}, T^{nb}; a_M)$ is symmetric with respect to $\widehat{a}_R$ and a_M is the center of $\mathcal{A}$, the best response of region A's median resident to b_R, denoted by $a_R^*(b_R)$, is (i) $a_R^*(b_R) = \bar{a}$ if $\widehat{a}_R \leq a_M$, and (ii) $a_R^*(b_R) = \underline{a}$ if $\widehat{a}_R \geq a_M$. After some calculation, we have that $\widehat{a}_R \leq a_M$ if and only if

$$\frac{(1+\gamma)(a_M(-1+\gamma)+b_R\gamma)}{3\gamma^2-1} \geq 0. \tag{25}$$

Because $0 \leq \gamma < 1/\sqrt{3}$, we have $3\gamma^2-1 < 0$. Hence, condition (25) is equivalent to $b_R \leq a_M(1-\gamma)/\gamma$. In addition, because $\mathcal{B} = [0,1]$, $a_M(1-\gamma)/\gamma$ belongs to $\mathcal{B}$ if and only if $a_M(1-\gamma)/\gamma \leq 1$ or $\gamma \geq a_M/(1+a_M)$. Summing up these observations, we reach Claim A1. ■

[1)]The calculations are performed using Mathematica; the code is available upon request.

The best response function of region B's median resident is the same as that in (12):

$$b_R^*(a_R) = \min\left\{1, \frac{1}{3} + \frac{a_R(1-\gamma)}{3\gamma}\right\}.$$

Considering two cases, we examine the Nash equilibrium.

Case (i) $0 \le \gamma \le \dfrac{a_M}{1+a_M}$

By Claim A1, resident a_M always selects $\bar{a}$ as the region A's representative. Substituting it into the best response function of region B's median resident, we have

$$b_R^*(\bar{a}) = \min\left\{1, \frac{1}{3} + \frac{1-\gamma}{3\gamma}\bar{a}\right\} = \min\left\{1, \frac{1-\gamma}{3\gamma}a_M + \frac{1+\gamma}{6\gamma}\right\},$$

because $\bar{a} = a_M + 1/2$.

Case (ii) $\dfrac{1}{\sqrt{3}} \ge \gamma \ge \dfrac{a_M}{1+a_M}$

Figure 1 summarizes the geometric relation between the best response functions of the median residents of regions A and B. The blue line represents the best response of region A's median resident, and the red line represents the best response of region B's median resident. In the figure, note that (ii-1) the slope of b_R^* is positive, a_R^* is discontinuous, and

$$\text{(ii-2) } b_R^*(\underline{a}) = \frac{a_M(1-\gamma)}{3\gamma} + \frac{-1+3\gamma}{6\gamma} < \frac{a_M(1-\gamma)}{\gamma} \text{ if } \frac{1}{\sqrt{3}} \ge \gamma \ge \frac{a_M}{1+a_M} \text{ and } a_M \ge \frac{1}{2}.$$

By (ii-1) and (ii-2), there is no Nash equilibrium in this case.

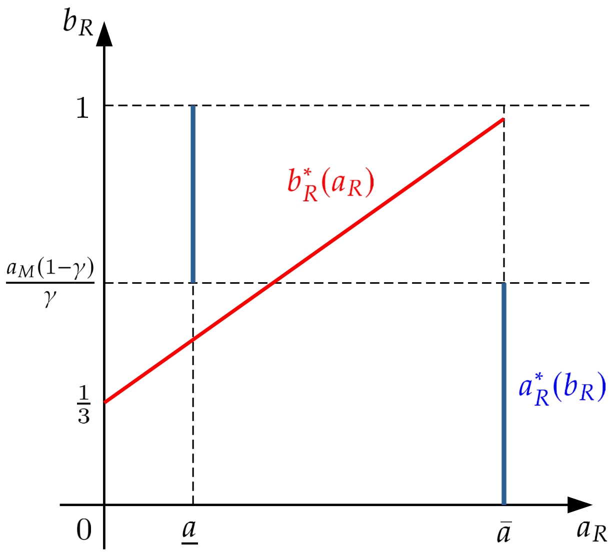

Figure 1: The case in which the second-order condition for region A's median resident does not hold

C Budget balance condition

In the main analysis, we have assumed that each region has enough income to meet any tax payment. Here, we relax this assumption. We still assume that x^E in (1) is feasible in the economy: $c(x^E) < I_A + I_B$. We discuss the importance of (i) the controllability of the breakdown level of the project, and (ii) the unbiased bargaining powers of the regions for the project to be completed efficiently.

When the negotiation breaks down, region A's representative $a_R \in \mathcal{A}$ chooses x^A to maximize his payoff $u(x^A, a_R) + (I_A/n_A) - (\gamma/n_A)c(x^A)$, subject to the budget conditions

$$\frac{\gamma}{n_A}c(x^A) \leq \frac{I_A}{n_A} \text{ and } \frac{1-\gamma}{n_B}c(x^A) \leq \frac{I_B}{n_B}.$$

The former (latter) is the budget condition for region A (region B, respectively). The cost of x^A is shared through the cost-matching grant of the central government. Thus, region B's budget condition constrains the decision of region A's representative. Here, x^A satisfies

$$u_x(x^A, a_R) = \frac{\gamma}{n_A}c'(x^A) \text{ if } c(x^A) \leq \min\left\{\frac{I_A}{\gamma}, \frac{I_B}{1-\gamma}\right\} \text{ and} \tag{26}$$

$$c(x^A) = \min\left\{\frac{I_A}{\gamma}, \frac{I_B}{1-\gamma}\right\} \text{ otherwise.} \tag{27}$$

Hereafter, without loss of generality, we assume that $I_A/\gamma = \min\{I_A/\gamma, I_B/(1-\gamma)\}$.

By (26), as long as $c(x^A) \leq I_A/\gamma$, x^A is increasing with respect to a_R. Hence, for all $\gamma \in [0,1]$, we can find the threshold $\tilde{a}_R(\gamma) \in \mathcal{A}$, such that

- for all $a_R \in [\underline{a}, \tilde{a}_R(\gamma)]$, $u_x(x^A, a_R) = \dfrac{\gamma}{n_A}c'(x^A)$, and
- for all $a_R \in (\tilde{a}_R(\gamma), \bar{a}]$, $c(x^A) = \dfrac{I_A}{\gamma}$.

Clearly, $\tilde{a}_R(\gamma)$ is increasing with respect to region A's income I_A. Furthermore, x^A is constant in a_R on the interval $(\tilde{a}_R(\gamma), \bar{a}]$. That is, region A cannot control the breakdown level of the project on this interval.

As in Section 3,

$$\bar{u}^A \equiv u(x^A, a_R) + \max\left\{0, \frac{I_A}{n_A} - \frac{\gamma}{n_A}c(x^A)\right\} \text{ and } \bar{v}^B \equiv v(x^A, b_R) + \max\left\{0, \frac{I_B}{n_B} - \frac{1-\gamma}{n_B}c(x^A)\right\} \tag{28}$$

are the breakdown payoffs to the representatives. The Nash bargaining problem is formulated as

$$\max_{x^{nb}, X_A^{nb}, X_B^{nb}} \beta \ln\left[u(x^{nb}, a_R) + X_A^{nb} - \bar{u}^A\right] + (1-\beta)\ln\left[v(x^{nb}, b_R) + X_B^{nb} - \bar{v}^B\right]$$

$$\text{subject to } I_A + I_B = n_A X_A^{nb} + n_B X_B^{nb} + c(x^{nb}),\ X_A^{nb} \geq 0, \text{ and } X_B^{nb} \geq 0,$$

where X_i^{nb} $(i = A, B)$ represents the per-capita consumption of private goods of region i, and the first constraint represents the resource constraint in the economy. As in Section 3, the Nash bargaining outcome depends on the types of the representatives. Then, we denote $x^{nb} = x^{nb}(a_R, b_R)$ and $X_i^{nb} = X_i^{nb}(a_R, b_R)$ $(i = A, B)$.

Result 2 *The Nash bargaining outcome is summarized as follows:*
Case 1. *If $X_i^{nb}(a_R, b_R) > 0$ for all $i \in \{A, B\}$, then*

$$X_A^{nb} = \frac{\beta}{n_A}\left(n_A u(x^{nb}, a_R) + n_B v(x^{nb}, b_R) + I_A + I_B - c(x^{nb}) - n_A\bar{u}^A - n_B\bar{v}^B\right) - \left(u(x^{nb}, a_R) - \bar{u}^A\right)$$

$$X_B^{nb} = \frac{1-\beta}{n_B}\left(n_A u(x^{nb}, a_R) + n_B v(x^{nb}, b_R) + I_A + I_B - c(x^{nb}) - n_A\bar{u}^A - n_B\bar{v}^B\right) - \left(v(x^{nb}, a_R) - \bar{v}^B\right),$$

where $x^{nb} = x^{nb}(a_R, b_R)$ and $x^A = x^A(a_R, \gamma)$ and $\bar{u}^A$ and $\bar{v}^B$ are determined as in (28).
Case 2. *If $X_i^{nb}(a_R, b_R) = 0$ and $X_j^{nb}(a_R, b_R) > 0$ $(i \neq j)$, then*

$$X_j^{nb}(a_R, b_R) = \frac{I_A + I_B - c(x^{nb})}{n_j}$$

where $x^{nb} = x^{nb}(a_R, b_R)$.

In both cases, x^{nb} *satisfies* $n_A u_x(x^{nb}, a_R) + n_B v_x(x^{nb}, b_R) = c'(x^{nb})$.[2)]

The value of β determines which of Cases 1 and 2 arises. In Case 1, the right-hand side of $X_i^{nb}(a_R, b_R)$ is not necessarily positive. If β is sufficiently small, then the right-hand side of $X_A^{nb}(a_R, b_R)$ may be negative.[3)] If β is sufficiently large, then the right-hand side of $X_B^{nb}(a_R, b_R)$ may be negative. Hence, $X_A^{nb}(a_R, b_R)$ and $X_B^{nb}(a_R, b_R)$ are both positive if and only if β is intermediate (the bargaining powers of two regions are not biased). Case 2 is observed if β is sufficiently large or small.

The payoffs to the median residents are $U(x^{nb}(a_R, b_R), X_A^{nb}(a_R, b_R); a_M) = u(x^{nb}(a_R, b_R), a_M) + X_A^{nb}(a_R, b_R)$ and $V(x^{nb}(a_R, b_R), X_B^{nb}(a_R, b_R); b_M) = v(x^{nb}(a_R, b_R), b_M) + X_B^{nb}(a_R, b_R)$.

In the following, to investigate the extensibility of our main result, we examine whether the project is undertaken efficiently through the self-representation.

Case 1 $X_A^{nb}(a_M, b_M), X_B^{nb}(a_M, b_M) > 0$

Suppose that the central government sets $\gamma = \gamma^*$, as in Theorem 1.

Case 1.1: If $a_M \le \tilde{a}_R(\gamma^*)$ holds, the payoffs to the median residents are the same as $U(x^{nb}, T^{nb}; a_M)$ and $V(x^{nb}, T^{nb}; b_M)$ in (6) and (7), respectively, in the main text. Thus, we have a similar result to that in Theorem 1.

Case 1.2: If $a_M > \tilde{a}_R(\gamma^*)$ holds, then the project cannot be undertaken efficiently under the grant with γ^*. Result 3 is crucial to showing this.

Result 3 *Suppose that* $a_M \in (\tilde{a}_R(\gamma^*), \bar{a}]$. *Then,* $(a_R, b_R) = (a_M, b_M)$ *satisfies the first-order condition* (8) *if and only if* $x^A(a_M, \gamma^*) = x^{nb}(a_M, b_M)$.[4)]

By Result 3, if the project level when the negotiation breaks down is the same as the efficient level $x^{nb}(a_M, b_M)$, then the self-representation can be an equilibrium. However, this is impossible because $c(x^A) = I_A/\gamma^*$ does not imply $x^A = x^{nb}(a_M, b_M)$ (see (27)). Thus, the cost-matching grant with γ^* does not achieve the efficient project.

Even if the cost-matching grant with γ^* is not effective, another cost-matching rate may achieve efficiency. Now, we consider the rate $\tilde{\gamma} = I_A/c(x^{nb}(a_M, b_M))$. Note that $c(x^A) = I_A/\tilde{\gamma}$ implies $x^A = x^{nb}(a_M, b_M)$. Similarly to Result 3, we can show that if a_M belongs to $(\tilde{a}(\tilde{\gamma}), \bar{a}]$, then the self-representation is an equilibrium such that the project is undertaken efficiently. Trivially, $\tilde{\gamma}$ does not work well to achieve efficiency without condition $a_M \in (\tilde{a}(\tilde{\gamma}), \bar{a}]$.

In conclusion, in Case 1.2, region A's income is insufficient, in that it cannot undertake the project so as to maximize the region A representative's surplus $u(x^A, a_M) - (\gamma^*/n_A)c(x^A)$. In this case, region A loses control of the breakdown level of the project around $(a_R, \gamma) = (a_M, \gamma^*)$. Thus, controlling the breakdown project level is crucial to the efficiency of the project. When region A's income is small, it would be useful to redistribute income to region A to restore the controllability.

Case 2 $X_i^{nb}(a_M, b_M) = 0$ and $X_j^{nb}(a_M, b_M) > 0$

We consider the case of $X_A^{nb}(a_M, b_M) = 0$ and $X_B^{nb}(a_M, b_M) > 0$. We examine the case in which the right-hand side of $X_A^{nb}(a_R, b_R)$ of Case 1 in Result 2 is negative at $(a_R, b_R) = (a_M, b_M)$:

$$0 > \frac{\beta}{n_A}\left(n_A u(x^{nb}, a_R) + n_B v(x^{nb}, b_R) + I_A + I_B - c(x^{nb}) - n_A \bar{u}^A - n_B \bar{v}^B\right) - \left(u(x^{nb}, a_R) - \bar{u}^A\right) \tag{29}$$

if $(a_R, b_R) = (a_M, b_M)$. The right-hand side of (29) is continuous in (a_R, b_R). Thus, we can find an open neighborhood of (a_M, b_M), denoted by $\mathcal{N}(a_M, b_M)$, such that for all $(a_R, b_R) \in \mathcal{N}(a_M, b_M)$,

[2)]The proof is at the end of this section.

[3)]Note that $u(x^{nb}, a_R) - \bar{u}^A$ and $v(x^{nb}, a_R) - \bar{v}^B$ are independent of β.

[4)]The proof is at the end of this section.

(29) is satisfied. By Case 2 of Result 2, the median residents' payoffs are

$$U(x^{nb}(a_R,b_R),X_A^{nb}(a_R,b_R);a_M) = u(x^{nb}(a_R,b_R),a_M)$$
$$\text{and } V(x^{nb}(a_R,b_R),X_B^{nb}(a_R,b_R);b_M) = v(x^{nb}(a_R,b_R),b_M) + \frac{I_A+I_B-c(x^{nb}(a_R,b_R))}{n_B}$$

in the neighborhood $\mathcal{N}(a_M,b_M)$. Because $x^{nb}(a_R,b_R)$ is increasing in a_R and the payoff to region A's median resident is increasing in x^{nb}, region A's median resident does not choose himself as the representative. In addition, the payoffs above do not depend on γ. Thus, it is impossible to control the median residents' behavior through the choice of γ. We obtain a similar result when $X_A^{nb}(a_M,b_M) > 0$ and $X_B^{nb}(a_M,b_M) = 0$.

In conclusion, when the bargaining powers of the regions are biased, the private good consumption of one of the regions may be zero. In this case, the project cannot be undertaken efficiently through self-representation. Therefore, when the budget conditions are introduced, an argument similar to that in Section 3 can be applied under nonbiased bargaining powers.

Proof of Result 2

The Lagrange function of the Nash bargaining problem is

$$\begin{aligned}\mathcal{L} = {}&\beta\ln\left[u(x,a_R)+X_A-\bar{u}^A\right]+(1-\beta)\ln\left[v(x,b_R)+X_B-\bar{v}^B\right]\\&+\lambda\left(I_A+I_B-n_AX_A-n_BX_B-c(x)\right)+\theta_AX_A+\theta_BX_B,\end{aligned}$$

where $\lambda,\theta_A,\theta_B \geq 0$. Differentiating $\mathcal{L}$ with respect to x, X_A, X_B, and λ yields

$$\frac{\beta u_x(x,a_R)}{u(x,a_R)+X_A-\bar{u}^A}+\frac{(1-\beta)v_x(x,b_R)}{v(x,b_R)+X_B-\bar{v}^B}-\lambda c'(x)=0, \tag{30}$$
$$\frac{\beta}{u(x,a_R)+X_A-\bar{u}^A}-\lambda n_A+\theta_A=0, \tag{31}$$
$$\frac{1-\beta}{v(x,b_R)+X_B-\bar{v}^B}-\lambda n_B+\theta_B=0, \tag{32}$$
$$I_A+I_B=n_AX_A+n_BX_B+c(x), \tag{33}$$

and, furthermore,

$$\theta_AX_A=0,\ \theta_BX_B=0,\ X_A\geq 0,\ \text{and}\ X_B\geq 0.$$

Case 1. $X_A, X_B > 0$ (i.e., $\theta_A=\theta_B=0$)
From (31) and (32),

$$\lambda=\frac{\beta}{n_A\left(u(x,a_R)+X_A-\bar{u}^A\right)}=\frac{1-\beta}{n_B\left(v(x,b_R)+X_B-\bar{v}^B\right)}. \tag{34}$$

Substituting this condition into (30) yields

$$\frac{1-\beta}{n_B\left(v(x,b_R)+X_B-\bar{v}^B\right)}\left(n_Au_x(x,a_R)+n_Bv_x(x,b_R)-c'(x)\right)=0,$$

which implies that $n_Au_x(x,a_R)+n_Bv_x(x,b_R)=c'(x)$. Again, from the second equality of (34),

$$X_B=\frac{(1-\beta)n_A\left(u(x,a_R)+X_A-\bar{u}^A\right)}{\beta n_B}-\left(v(x,b_R)-\bar{u}^B\right).$$

Substituting this into (33) yields

$$X_A=\frac{\beta}{n_A}\left(n_Au(x,a_R)+n_Bv(x,b_R)+I_A+I_B-c(x)-n_A\bar{u}^A-n_B\bar{v}^B\right)-\left(u(x,a_R)-\bar{u}^A\right).$$

Then,

$$X_B = \frac{1-\beta}{n_B}\left(n_A u(x,a_R) + n_B v(x,b_R) + I_A + I_B - c(x) - n_A \bar{u}^A - n_B \bar{v}^B\right) - \left(v(x,a_R) - \bar{v}^B\right).$$

Case 2. $X_i = 0$ and $X_j > 0$ (i.e., $\theta_j = 0$)
We consider the case in which $X_A = 0$ and $X_B > 0$.[5] From (32),

$$\lambda = \frac{1-\beta}{n_B\left(v(x,b_R) + X_B - \bar{v}^B\right)}. \tag{35}$$

From (33),

$$X_B = \frac{I_A + I_B - c(x)}{n_B}.$$

Because $X_B > 0$, $I_A + I_B > c(x)$ holds. From (31) and (35),

$$\frac{\beta}{u(x,a_R) + X_A - \bar{u}^A} = \frac{n_A(1-\beta)}{n_B\left(v(x,b_R) + X_B - \bar{v}^B\right)} - \theta_A. \tag{36}$$

Substituting (35) and (36) into (30) yields

$$\frac{1-\beta}{n_B\left(v(x,b_R) + X_B - \bar{v}^B\right)}\left(n_A u_x(x,a_R) + n_B v_x(x,b_R) - c'(x)\right) = \theta_A u_x(x,a_R). \tag{37}$$

We show that $\theta_A = 0$ by contradiction. Suppose, to the contrary, that $\theta_A > 0$. Then, from (37), we have that $n_A u_x(x,a_R) + n_B v_x(x,b_R) > c'(x)$. We can take another project level $\tilde{x}$, which is slightly greater than x, such that $\tilde{x} > x$, $n_A u_x(\tilde{x},a_R) + n_B v_x(\tilde{x},b_R) > c'(\tilde{x})$, and $I_A + I_B > c(\tilde{x})$. Set

$$\tilde{X}_A = -u(\tilde{x},a_R) + u(x,a_R) + \varepsilon_A \quad \text{and} \quad \tilde{X}_B = -v(\tilde{x},b_R) + v(x,b_R) + \frac{I_A + I_B - c(x)}{n_B} + \varepsilon_B,$$

where

$$\begin{aligned}
&\varepsilon_A \geq u(\tilde{x},a_R) - u(x,a_R),\\
&\varepsilon_B \geq \max\left\{0, v(\tilde{x},b_R) - v(x,b_R) - \frac{I_A + I_B - c(x)}{n_B}\right\}, \text{ and}\\
n_A\varepsilon_A + n_B\varepsilon_B &\leq I_A + I_B - n_A\left(-u(\tilde{x},a_R) + u(x,a_R)\right) - n_B\left(-v(\tilde{x},b_R) + v(x,b_R) + \frac{I_A + I_B - c(x)}{n_B}\right)\\
&\quad - c(\tilde{x})
\end{aligned} \tag{38}$$

(38) implies that $n_A\tilde{X}_A + n_B\tilde{X}_B + c(\tilde{x}) \leq I_A + I_B$: $(\tilde{x}, \tilde{X}_A, \tilde{X}_B)$ is feasible in this economy.

At (x, X_A, X_B), $U(x, X_A; a_R) = u(x,a_R)$ and $V(x, X_B; b_R) = v(x,b_R) + (I_A + I_B - c(x))/n_B$. At $(\tilde{x}, \tilde{X}_A, \tilde{X}_B)$, $U(\tilde{x}, \tilde{X}_A; a_R) = u(\tilde{x},a_R) + \tilde{X}_A = u(x,a_R) + \varepsilon_A$ and $V(\tilde{x}, \tilde{X}_B; b_R) = v(\tilde{x},b_R) + \tilde{X}_B = v(x,b_R) + (I_A + I_B - c(x))/n_B + \varepsilon_B$. Because $\varepsilon_A > 0$ and $\varepsilon_B \geq 0$, (x, X_A, X_B) does not maximize the Nash product function, which is a contradiction. Thus, $\theta_A = 0$ and, hence, from (37), $n_A u_x(x,a_R) + n_B v_x(x,b_R) = c'(x)$.

Case 3. $X_A = X_B = 0$
From (33), the project level is determined by $c(x) = I_A + I_B$. ■

[5]The case of $X_A > 0$ and $X_B = 0$ is similar.

Proof of Result 3

Because $a_M \in (\tilde{a}_R(\gamma^*), \bar{a}]$ and x^A is constant on this interval, by (27), we have

$$\left.\frac{\partial x^A}{\partial a_R}\right|_{a_R=a_M,\gamma=\gamma^*} = 0.$$

As in Lemma 2, if $(a_R, b_R) = (a_M, b_M)$ and $\gamma = \gamma^*$, then

$$\begin{aligned}\frac{\partial\, U(x^{nb}, X_A^{nb}; a_M)}{\partial\, a_R} =& \frac{\partial x^{nb}}{\partial a_R}\left[u_x(x^{nb}(a_M, b_M), a_M) - u_x(x^{nb}(a_M, b_M), a_M)\right] \\ &-(1-\beta)\left[u_a(x^{nb}(a_M, b_M), a_M) - u_a(x^A(a_M, \gamma^*), a_M)\right] \\ =& -(1-\beta)\left[u_a(x^{nb}(a_M, b_M), a_M) - u_a(x^A(a_M, \gamma^*), a_M)\right]\end{aligned}$$

and

$$\begin{aligned}\frac{\partial\, V(x^{nb}, X_B^{nb}; b_M)}{\partial\, b_R} =& \frac{\partial x^{nb}}{\partial b_R}[v_x(x^{nb}(a_M, b_M), b_M) - v_x(x^{nb}(a_M, b_M), b_M)] \\ &- \beta[v_b(x^{nb}(a_M, b_M), b_M) - v_b(x^A(a_M, \gamma^*), b_M)] \\ =& -\beta[v_b(x^{nb}(a_M, b_M), b_M) - v_b(x^A(a_M, \gamma^*), b_M)].\end{aligned}$$

Thus, the first-order condition of (8) is satisfied at $(a_R, b_R) = (a_M, b_M)$ and $\gamma = \gamma^*$ if and only if $x^{nb}(a_M, b_M) = x^A(a_M, \gamma^*)$. ■

D A model of the endogenous choice of γ

We analyze the model presented in Section 4.2 in the main text. Without loss of generality, we assume that $n_A > n_B$, which implies that the value of γ is decided by region A's representative in the legislature of the central government. Hereafter, we explicitly denote T^{nb} by $T^{nb}(\gamma)$, and x^{nb} by $x^{nb}(\gamma)$ because they depend on the value of γ.

First, we show that region A's median resident is decisive in determining γ. The value of γ depends on who is the representative of region A. Let $\gamma[a_M]$ be the cost-matching rate that maximizes the payoff to resident a_M, $U(x^{nb}(\gamma), T^{nb}(\gamma); a_M) = a_M\mu(x^{nb}(\gamma)) + (I_A/n_A) - (\gamma/n_A)c(x^{nb}(\gamma)) + (n_B/n_A)T^{nb}(\gamma)$. Let $\gamma' \neq \gamma[a_M]$. Because $\gamma[a_M]$ maximizes the payoff of a_M,

$$\begin{aligned}&a_M\mu\left(x^{nb}\left(\gamma[a_M]\right)\right) + \left(\frac{I_A}{n_A}\right) - \left(\frac{\gamma[a_M]}{n_A}\right)c\left(x^{nb}\left(\gamma[a_M]\right)\right) + \left(\frac{n_B}{n_A}\right)T^{nb}\left(\gamma[a_M]\right) \\ \geq& a_M\mu\left(x^{nb}\left(\gamma'\right)\right) + \left(\frac{I_A}{n_A}\right) - \left(\frac{\gamma'}{n_A}\right)c\left(x^{nb}\left(\gamma'\right)\right) + \left(\frac{n_B}{n_A}\right)T^{nb}\left(\gamma'\right),\end{aligned}$$

which is equivalent to

$$\begin{aligned}&a_M\left[\mu\left(x^{nb}\left(\gamma[a_M]\right)\right) - \mu\left(x^{nb}\left(\gamma'\right)\right)\right] \\ \geq& \frac{1}{n_A}\left[\gamma c\left(x^{nb}\left(\gamma[a_M]\right)\right) - \gamma' c\left(x^{nb}\left(\gamma'\right)\right)\right] + \frac{n_B}{n_A}\left[T^{nb}\left(\gamma'\right) - T^{nb}\left(\gamma[a_M]\right)\right].\end{aligned} \tag{39}$$

If $x^{nb}\left(\gamma[a_M]\right) > x^{nb}\left(\gamma'\right)$, then for all $a' \in \mathcal{A}$, such that $a' > a_M$,

$$a_M\left[\mu\left(x^{nb}\left(\gamma[a_M]\right)\right) - \mu\left(x^{nb}\left(\gamma'\right)\right)\right] < a'\left[\mu\left(x^{nb}\left(\gamma[a_M]\right)\right) - \mu\left(x^{nb}\left(\gamma'\right)\right)\right]. \tag{40}$$

Combining (39) and (40) yields

$$\begin{aligned}&a'\mu\left(x^{nb}\left(\gamma[a_M]\right)\right) + \left(\frac{I_A}{n_A}\right) - \left(\frac{\gamma[a_M]}{n_A}\right)c\left(x^{nb}\left(\gamma[a_M]\right)\right) + \left(\frac{n_B}{n_A}\right)T^{nb}\left(\gamma[a_M]\right) \\ >& a'\mu\left(x^{nb}\left(\gamma'\right)\right) + \left(\frac{I_A}{n_A}\right) - \left(\frac{\gamma'}{n_A}\right)c\left(x^{nb}\left(\gamma'\right)\right) + \left(\frac{n_B}{n_A}\right)T^{nb}\left(\gamma'\right).\end{aligned} \tag{41}$$

In a similar way, we obtain (41) for all $a' \in \mathcal{A}$, such that $a' < a_M$ if $x^{nb}(\gamma[a_M]) < x^{nb}(\gamma')$. Therefore, these observations prove that γ chosen by a_M is the pairwise majority winner.[6)]

Because $\gamma[a_M]$ is chosen by the median resident a_M and $\gamma[a_M]$ is the pairwise majority winner among $\gamma \in [0, 1]$, then the majority of region A's residents must support the median resident a_M as the representative of the central legislature.[7)] In conclusion, the median resident of more populous region is decisive in determining γ.

We create Table 1 based on the numerical analyses in Example 1, taking $(n_B, b_M, \beta) = (1, 0.5, 0.5)$ as fixed. Because the second-order condition for region A's median resident holds if $1/\sqrt{3} \approx 0.577 < \gamma \leq 1$ (see (11)), we derive the optimal γ for region A's median resident constrained on the interval $(1/\sqrt{3}, 1]$. Table 1 shows the relation between n_A, a_M, $\gamma[a_M]$, and γ^* (the Lindahl price in Theorem 1). See Table 1 and the discussion after Table 1 in the main text.

[6)]Similarly, we can show that γ chosen by b_M is the pairwise majority winner if $n_B > n_A$.

[7)]We assume that if the same outcomes (i.e., $x^{nb}(\gamma[a_M])$ and $T^{nb}(\gamma[a_M])$) are achieved when $a \neq a_M$ is region A's representative, then a_M is majority-preferred to a.