Mathematica code: R. Shinohara "Interregional Negotiations and Strategic Delegation under Government Subsidy Schemes"

Payoff functions to regional residents

We consider a situation in which each resident has a linear benefit function u(x,a)=ax and v(x,b)=bx and the cost function is c(x)=$\frac{1}{\alpha} x^{\alpha}(\alpha \geq 2)$. In the following, the first (second) line defines the payoff to a resident in region A (region B, respectively).

$$uA[x\_, a\_, T\_, \alpha\_] := a\, x - \frac{\gamma}{n_A * \alpha} * x^{\alpha} + \frac{n_B}{n_A} * T$$

$$uB[x\_, b\_, T\_, \alpha\_] := b\, x - \frac{(1-\gamma)}{n_B\, \alpha} * x^{\alpha} - T$$

Solving the model in Section 3 backward

Bargaining breakdown outcome

First, we derive the breakdown level of the project as follows:

$$\text{Solve}[D[uA[x, a_R, 0, \alpha], x] == 0, x]$$

$$\left\{\left\{x \to \left(\frac{a_R\, n_A}{\gamma}\right)^{\frac{1}{-1+\alpha}}\right\}\right\}$$

Thus, $x^A(a_R, \gamma) = \left(\frac{a_R\, n_A}{\gamma}\right)^{\frac{1}{-1+\alpha}}$. The first derivative of $x^A(a_R, \gamma)$ is $\frac{\left(\frac{a_R\, n_A}{\gamma}\right)^{\frac{1}{-1+\alpha}}}{(-1+\alpha)\, a_R}$:

$$\text{Simplify}\left[D\left[\left(\frac{a_R\, n_A}{\gamma}\right)^{\frac{1}{-1+\alpha}}, a_R\right], \alpha \geq 2\right]$$

$$\frac{\left(\frac{a_R\, n_A}{\gamma}\right)^{\frac{1}{-1+\alpha}}}{(-1+\alpha)\, a_R}$$

The manipulability of breakdown outcome can be measured by $\frac{d^2 x^A}{da_R\, d\gamma} = \frac{\left(\frac{a_R\, n_A}{\gamma}\right)^{\frac{1}{-1+\alpha}}}{(-1+\alpha)\, a_R}$, which is shown to be negative as follows:

$$\text{Simplify}\left[D\left[\frac{\left(\frac{a_R\, n_A}{\gamma}\right)^{\frac{1}{-1+\alpha}}}{(-1+\alpha)\, a_R}, \gamma\right]\right]$$

$$-\frac{\left(\frac{a_R\, n_A}{\gamma}\right)^{\frac{1}{-1+\alpha}}}{(-1+\alpha)^2\, \gamma\, a_R}$$

Nash bargaining outcome

The Nash product is given as NP in the next line:

$$NP = \beta\, \mathrm{Log}\left[uA[x, aR, T, \alpha] - uA\left[\left(\frac{aR\, n_A}{\gamma}\right)^{\frac{1}{-1+\alpha}}, aR, 0, \alpha\right]\right] + (1-\beta)\, \mathrm{Log}\left[uB[x, bR, T, \alpha] - uB\left[\left(\frac{aR\, n_A}{\gamma}\right)^{\frac{1}{-1+\alpha}}, bR, 0, \alpha\right]\right]$$

$$(1-\beta)\, \mathrm{Log}\left[-T + bR\, x - bR\left(\frac{aR\, n_A}{\gamma}\right)^{\frac{1}{-1+\alpha}} - \frac{x^{\alpha}(1-\gamma)}{\alpha\, n_B} + \frac{(1-\gamma)\left(\left(\frac{aR\, n_A}{\gamma}\right)^{\frac{1}{-1+\alpha}}\right)^{\alpha}}{\alpha\, n_B}\right] + \beta\, \mathrm{Log}\left[aR\, x - \frac{x^{\alpha}\gamma}{\alpha\, n_A} - aR\left(\frac{aR\, n_A}{\gamma}\right)^{\frac{1}{-1+\alpha}} + \frac{\gamma\left(\left(\frac{aR\, n_A}{\gamma}\right)^{\frac{1}{-1+\alpha}}\right)^{\alpha}}{\alpha\, n_A} + \frac{T\, n_B}{n_A}\right]$$

In the next two lines, we differentiate NP with respect to x and T:

```
D[NP, x]
D[NP, T]
```

$$\left((1-\beta)\left(bR - \frac{x^{-1+\alpha}(1-\gamma)}{n_B}\right)\right) \Big/ \left(-T + bR\, x - bR\left(\frac{aR\, n_A}{\gamma}\right)^{\frac{1}{-1+\alpha}} - \frac{x^{\alpha}(1-\gamma)}{\alpha\, n_B} + \frac{(1-\gamma)\left(\left(\frac{aR\, n_A}{\gamma}\right)^{\frac{1}{-1+\alpha}}\right)^{\alpha}}{\alpha\, n_B}\right) + \left(\beta\left(aR - \frac{x^{-1+\alpha}\gamma}{n_A}\right)\right) \Big/ \left(aR\, x - \frac{x^{\alpha}\gamma}{\alpha\, n_A} - aR\left(\frac{aR\, n_A}{\gamma}\right)^{\frac{1}{-1+\alpha}} + \frac{\gamma\left(\left(\frac{aR\, n_A}{\gamma}\right)^{\frac{1}{-1+\alpha}}\right)^{\alpha}}{\alpha\, n_A} + \frac{T\, n_B}{n_A}\right)$$

$$-\left((1-\beta) \Big/ \left(-T + bR\, x - bR\left(\frac{aR\, n_A}{\gamma}\right)^{\frac{1}{-1+\alpha}} - \frac{x^{\alpha}(1-\gamma)}{\alpha\, n_B} + \frac{(1-\gamma)\left(\left(\frac{aR\, n_A}{\gamma}\right)^{\frac{1}{-1+\alpha}}\right)^{\alpha}}{\alpha\, n_B}\right)\right) + (\beta\, n_B) \Big/ \left(n_A\left(aR\, x - \frac{x^{\alpha}\gamma}{\alpha\, n_A} - aR\left(\frac{aR\, n_A}{\gamma}\right)^{\frac{1}{-1+\alpha}} + \frac{\gamma\left(\left(\frac{aR\, n_A}{\gamma}\right)^{\frac{1}{-1+\alpha}}\right)^{\alpha}}{\alpha\, n_A} + \frac{T\, n_B}{n_A}\right)\right)$$

The project level achieved through the Nash bargaining is calculated as $x^{nb}(aR, bR) = (aR\, n_A + bR\, n_B)^{\frac{1}{-1+\alpha}}$.

$$\mathrm{Solve}[n_A * aR + n_B * bR == x\string^(\alpha - 1), x]$$

$$\left\{\left\{x \to (aR\, n_A + bR\, n_B)^{\frac{1}{-1+\alpha}}\right\}\right\}$$

In the next lines, the transfer level is calculated:

```
Solve[D[NP, T] == 0, T]
```

$$\left\{\left\{T \to -\frac{1}{\alpha\, n_B}\left(x^{\alpha}\beta - x^{\alpha}\gamma + aR\, x\, \alpha\, n_A - aR\, x\, \alpha\, \beta\, n_A - aR\, \alpha\, n_A\left(\frac{aR\, n_A}{\gamma}\right)^{\frac{1}{-1+\alpha}} + aR\, \alpha\, \beta\, n_A\left(\frac{aR\, n_A}{\gamma}\right)^{\frac{1}{-1+\alpha}} - \beta\left(\left(\frac{aR\, n_A}{\gamma}\right)^{\frac{1}{-1+\alpha}}\right)^{\alpha} + \gamma\left(\left(\frac{aR\, n_A}{\gamma}\right)^{\frac{1}{-1+\alpha}}\right)^{\alpha} - bR\, x\, \alpha\, \beta\, n_B + bR\, \alpha\, \beta\left(\frac{aR\, n_A}{\gamma}\right)^{\frac{1}{-1+\alpha}} n_B\right)\right\}\right\}$$

$$-\frac{1}{\alpha\, n_B}\left(x^\alpha \beta - x^\alpha \gamma + aR\, x\, \alpha\, n_A - aR\, x\, \alpha\, \beta\, n_A - aR\, \alpha\, n_A \left(\frac{aR\, n_A}{\gamma}\right)^{\frac{1}{-1+\alpha}} + aR\, \alpha\, \beta\, n_A \left(\frac{aR\, n_A}{\gamma}\right)^{\frac{1}{-1+\alpha}} - \beta\left(\left(\frac{aR\, n_A}{\gamma}\right)^{\frac{1}{-1+\alpha}}\right)^\alpha + \gamma\left(\left(\frac{aR\, n_A}{\gamma}\right)^{\frac{1}{-1+\alpha}}\right)^\alpha - bR\, x\, \alpha\, \beta\, n_B + bR\, \alpha\, \beta \left(\frac{aR\, n_A}{\gamma}\right)^{\frac{1}{-1+\alpha}} n_B\right) \text{ /. } x \text{ -> } (aR\, n_A + bR\, n_B)^{\frac{1}{-1+\alpha}}$$

```
Simplify[
 %]
```

$$-\frac{1}{\alpha\, n_B}\left(-aR\, \alpha\, n_A \left(\frac{aR\, n_A}{\gamma}\right)^{\frac{1}{-1+\alpha}} + aR\, \alpha\, \beta\, n_A \left(\frac{aR\, n_A}{\gamma}\right)^{\frac{1}{-1+\alpha}} - \beta\left(\left(\frac{aR\, n_A}{\gamma}\right)^{\frac{1}{-1+\alpha}}\right)^\alpha + \gamma\left(\left(\frac{aR\, n_A}{\gamma}\right)^{\frac{1}{-1+\alpha}}\right)^\alpha + bR\, \alpha\, \beta \left(\frac{aR\, n_A}{\gamma}\right)^{\frac{1}{-1+\alpha}} n_B + aR\, \alpha\, n_A\, (aR\, n_A + bR\, n_B)^{\frac{1}{-1+\alpha}} - aR\, \alpha\, \beta\, n_A\, (aR\, n_A + bR\, n_B)^{\frac{1}{-1+\alpha}} - bR\, \alpha\, \beta\, n_B\, (aR\, n_A + bR\, n_B)^{\frac{1}{-1+\alpha}} + \beta\left((aR\, n_A + bR\, n_B)^{\frac{1}{-1+\alpha}}\right)^\alpha - \gamma\left((aR\, n_A + bR\, n_B)^{\frac{1}{-1+\alpha}}\right)^\alpha\right)$$

$$\frac{1}{\alpha\, n_B}\left(\alpha\, \gamma \left(\frac{aR\, n_A}{\gamma}\right)^{\frac{\alpha}{-1+\alpha}} - \alpha\, \beta\, \gamma \left(\frac{aR\, n_A}{\gamma}\right)^{\frac{\alpha}{-1+\alpha}} + \beta\left(\left(\frac{aR\, n_A}{\gamma}\right)^{\frac{1}{-1+\alpha}}\right)^\alpha - \gamma\left(\left(\frac{aR\, n_A}{\gamma}\right)^{\frac{1}{-1+\alpha}}\right)^\alpha + aR\, \alpha\, (-1+\beta)\, n_A\, (aR\, n_A + bR\, n_B)^{\frac{1}{-1+\alpha}} - \beta\left((aR\, n_A + bR\, n_B)^{\frac{1}{-1+\alpha}}\right)^\alpha + \gamma\left((aR\, n_A + bR\, n_B)^{\frac{1}{-1+\alpha}}\right)^\alpha - bR\, \alpha\, \beta\, n_B\left(\left(\frac{aR\, n_A}{\gamma}\right)^{\frac{1}{-1+\alpha}} - (aR\, n_A + bR\, n_B)^{\frac{1}{-1+\alpha}}\right)\right)$$

By the calculations above, we define the negotiated level of project Xnb[aR, bR, *α*] and transfers Tnb[aR, bR, *β*, *γ*, *α*] as follows:

$$\text{Xnb[aR\_, bR\_, }\alpha\text{\_] := } (aR\, n_A + bR\, n_B)^{\frac{1}{-1+\alpha}}$$

$$\text{Tnb[aR\_, bR\_, }\beta\text{\_, }\gamma\text{\_, }\alpha\text{\_] := }$$

$$\frac{1}{\alpha\, n_B}\left(\alpha\, \gamma \left(\frac{aR\, n_A}{\gamma}\right)^{\frac{\alpha}{-1+\alpha}} - \alpha\, \beta\, \gamma \left(\frac{aR\, n_A}{\gamma}\right)^{\frac{\alpha}{-1+\alpha}} + \beta\left(\left(\frac{aR\, n_A}{\gamma}\right)^{\frac{1}{-1+\alpha}}\right)^\alpha - \gamma\left(\left(\frac{aR\, n_A}{\gamma}\right)^{\frac{1}{-1+\alpha}}\right)^\alpha + aR\, \alpha\, (-1+\beta)\, n_A\, (aR\, n_A + bR\, n_B)^{\frac{1}{-1+\alpha}} - \beta\left((aR\, n_A + bR\, n_B)^{\frac{1}{-1+\alpha}}\right)^\alpha + \gamma\left((aR\, n_A + bR\, n_B)^{\frac{1}{-1+\alpha}}\right)^\alpha - bR\, \alpha\, \beta\, n_B\left(\left(\frac{aR\, n_A}{\gamma}\right)^{\frac{1}{-1+\alpha}} - (aR\, n_A + bR\, n_B)^{\frac{1}{-1+\alpha}}\right)\right)$$

In the next lines, the first (second) line calculates the payoff to the median resident of region A (region B, respectively).

```
uA[Xnb[aR, bR, α], aM, Tnb[aR, bR, β, γ, α], α]
uB[Xnb[aR, bR, α], bM, Tnb[aR, bR, β, γ, α], α]
```

$$\mathrm{aM}\,(\mathrm{aR}\,n_A+\mathrm{bR}\,n_B)^{\frac{1}{-1+\alpha}}-\frac{\gamma\left((\mathrm{aR}\,n_A+\mathrm{bR}\,n_B)^{\frac{1}{-1+\alpha}}\right)^{\alpha}}{\alpha\,n_A}+\frac{1}{\alpha\,n_A}\left(\alpha\,\gamma\left(\frac{\mathrm{aR}\,n_A}{\gamma}\right)^{\frac{\alpha}{-1+\alpha}}-\alpha\,\beta\,\gamma\left(\frac{\mathrm{aR}\,n_A}{\gamma}\right)^{\frac{\alpha}{-1+\alpha}}+\beta\left(\left(\frac{\mathrm{aR}\,n_A}{\gamma}\right)^{\frac{1}{-1+\alpha}}\right)^{\alpha}-\gamma\left(\left(\frac{\mathrm{aR}\,n_A}{\gamma}\right)^{\frac{1}{-1+\alpha}}\right)^{\alpha}+\mathrm{aR}\,\alpha\,(-1+\beta)\,n_A\,(\mathrm{aR}\,n_A+\mathrm{bR}\,n_B)^{\frac{1}{-1+\alpha}}-\beta\left((\mathrm{aR}\,n_A+\mathrm{bR}\,n_B)^{\frac{1}{-1+\alpha}}\right)^{\alpha}+\gamma\left((\mathrm{aR}\,n_A+\mathrm{bR}\,n_B)^{\frac{1}{-1+\alpha}}\right)^{\alpha}-\mathrm{bR}\,\alpha\,\beta\,n_B\left(\left(\frac{\mathrm{aR}\,n_A}{\gamma}\right)^{\frac{1}{-1+\alpha}}-(\mathrm{aR}\,n_A+\mathrm{bR}\,n_B)^{\frac{1}{-1+\alpha}}\right)\right)$$

$$\mathrm{bM}\,(\mathrm{aR}\,n_A+\mathrm{bR}\,n_B)^{\frac{1}{-1+\alpha}}-\frac{(1-\gamma)\left((\mathrm{aR}\,n_A+\mathrm{bR}\,n_B)^{\frac{1}{-1+\alpha}}\right)^{\alpha}}{\alpha\,n_B}-\frac{1}{\alpha\,n_B}\left(\alpha\,\gamma\left(\frac{\mathrm{aR}\,n_A}{\gamma}\right)^{\frac{\alpha}{-1+\alpha}}-\alpha\,\beta\,\gamma\left(\frac{\mathrm{aR}\,n_A}{\gamma}\right)^{\frac{\alpha}{-1+\alpha}}+\beta\left(\left(\frac{\mathrm{aR}\,n_A}{\gamma}\right)^{\frac{1}{-1+\alpha}}\right)^{\alpha}-\gamma\left(\left(\frac{\mathrm{aR}\,n_A}{\gamma}\right)^{\frac{1}{-1+\alpha}}\right)^{\alpha}+\mathrm{aR}\,\alpha\,(-1+\beta)\,n_A\,(\mathrm{aR}\,n_A+\mathrm{bR}\,n_B)^{\frac{1}{-1+\alpha}}-\beta\left((\mathrm{aR}\,n_A+\mathrm{bR}\,n_B)^{\frac{1}{-1+\alpha}}\right)^{\alpha}+\gamma\left((\mathrm{aR}\,n_A+\mathrm{bR}\,n_B)^{\frac{1}{-1+\alpha}}\right)^{\alpha}-\mathrm{bR}\,\alpha\,\beta\,n_B\left(\left(\frac{\mathrm{aR}\,n_A}{\gamma}\right)^{\frac{1}{-1+\alpha}}-(\mathrm{aR}\,n_A+\mathrm{bR}\,n_B)^{\frac{1}{-1+\alpha}}\right)\right)$$

The calculation of equilibria when $\alpha = 2$ (quadratic cost)

The project level and the transfer level are calculated as follows:

```
Xnb[aR, bR, 2]
Simplify[Tnb[aR, bR, β, γ, 2]]
```

$$\mathrm{aR}\,n_A+\mathrm{bR}\,n_B$$

$$\frac{(\beta+\gamma)\,(\mathrm{aR}\,(-1+\gamma)\,n_A+\mathrm{bR}\,\gamma\,n_B)^2}{2\,\gamma^2\,n_B}$$

The payoff toregion A’s median resident is derived in the next line.

```
Collect[uA[Xnb[aR, bR, 2], aM, ((β + γ) (aR (-1 + γ) nA + bR γ nB)^2)/(2 γ^2 nB), 2], aR]
```

$$\mathrm{aR}^2\left(-\frac{\gamma\,n_A}{2}+\frac{(-1+\gamma)^2\,(\beta+\gamma)\,n_A}{2\,\gamma^2}\right)+\mathrm{aM}\,\mathrm{bR}\,n_B-\frac{\mathrm{bR}^2\,\gamma\,n_B^2}{2\,n_A}+\frac{\mathrm{bR}^2\,(\beta+\gamma)\,n_B^2}{2\,n_A}+\mathrm{aR}\left(\mathrm{aM}\,n_A-\mathrm{bR}\,\gamma\,n_B+\frac{\mathrm{bR}\,(-1+\gamma)\,(\beta+\gamma)\,n_B}{\gamma}\right)$$

$$D\left[uA\left[Xnb[aR, bR, 2], aM, \frac{(\beta+\gamma)\ (aR\ (-1+\gamma)\ n_A + bR\,\gamma\, n_B)^2}{2\,\gamma^2\, n_B}, 2\right], aR\right]$$

$$aM\, n_A - \gamma\ (aR\, n_A + bR\, n_B) + \frac{1}{\gamma^2}(-1+\gamma)\ (\beta+\gamma)\ (aR\ (-1+\gamma)\ n_A + bR\,\gamma\, n_B)$$

In the next line, the best response function for region A's median resident is derived.

$$Solve\left[D\left[uA\left[Xnb[aR, bR, 2], aM, \frac{(\beta+\gamma)\ (aR\ (-1+\gamma)\ n_A + bR\,\gamma\, n_B)^2}{2\,\gamma^2\, n_B}, 2\right], aR\right] == 0, aR\right]$$

$$\left\{\left\{aR \to -\left((\gamma\ (aM\,\gamma\, n_A - bR\,\beta\, n_B - bR\,\gamma\, n_B + bR\,\beta\,\gamma\, n_B))\ /\ \left(\left(\beta+\gamma-2\,\beta\,\gamma-2\,\gamma^2+\beta\,\gamma^2\right)\ n_A\right)\right)\right\}\right\}$$

In the next line, the best response function for region B's median resident is derived.

$$Solve\left[D\left[uB\left[Xnb[aR, bR, 2], bM, \frac{(\beta+\gamma)\ (aR\ (-1+\gamma)\ n_A + bR\,\gamma\, n_B)^2}{2\,\gamma^2\, n_B}, 2\right], bR\right] == 0, bR\right]$$

$$\left\{\left\{bR \to \frac{aR\,\beta\, n_A - aR\,\beta\,\gamma\, n_A + bM\,\gamma\, n_B}{(1+\beta)\ \gamma\, n_B}\right\}\right\}$$

The equilibrium representatives are calculated in the next line.

$$Solve\left[\left\{aR == -\left((\gamma\ (aM\,\gamma\, n_A - bR\,\beta\, n_B - bR\,\gamma\, n_B + bR\,\beta\,\gamma\, n_B))\ /\ \left(\left(\beta+\gamma-2\,\beta\,\gamma-2\,\gamma^2+\beta\,\gamma^2\right)\ n_A\right)\right),\ bR == \frac{aR\,\beta\, n_A - aR\,\beta\,\gamma\, n_A + bM\,\gamma\, n_B}{(1+\beta)\ \gamma\, n_B}\right\}, \{aR, bR\}\right]$$

```
Simplify[
 %]
```

$$\left\{\left\{aR \to -\left(\left(-aM\,\gamma^2\, n_A - aM\,\beta\,\gamma^2\, n_A + bM\,\beta\,\gamma\, n_B + bM\,\gamma^2\, n_B - bM\,\beta\,\gamma^2\, n_B\right)\ /\ ((\beta+\gamma)\ (-1+2\,\gamma)\ n_A)\right),\ bR \to -\left(\left(-aM\,\beta\,\gamma\, n_A + aM\,\beta\,\gamma^2\, n_A + bM\,\beta\, n_B + bM\,\gamma\, n_B - 2\,bM\,\beta\,\gamma\, n_B - 2\,bM\,\gamma^2\, n_B + bM\,\beta\,\gamma^2\, n_B\right)\ /\ ((\beta+\gamma)\ (-1+2\,\gamma)\ n_B)\right)\right\}\right\}$$

$$\left\{\left\{aR \to (\gamma\ (aM\ (1+\beta)\ \gamma\, n_A + bM\ (\beta\ (-1+\gamma) - \gamma)\ n_B))\ /\ ((\beta+\gamma)\ (-1+2\,\gamma)\ n_A),\ bR \to -\left(\left(aM\,\beta\ (-1+\gamma)\ \gamma\, n_A + bM\left(\beta\ (-1+\gamma)^2 + \gamma - 2\,\gamma^2\right)\ n_B\right)\ /\ ((\beta+\gamma)\ (-1+2\,\gamma)\ n_B)\right)\right\}\right\}$$

By using the equilibrium representatives, the equilibrium level of the project and the equilibrium level of the transfers are derived in the next line.

$$Simplify\left[Xnb\left[(\gamma\ (aM\ (1+\beta)\ \gamma\, n_A + bM\ (\beta\ (-1+\gamma) - \gamma)\ n_B))\ /\ ((\beta+\gamma)\ (-1+2\,\gamma)\ n_A),\ -\left(\left(aM\,\beta\ (-1+\gamma)\ \gamma\, n_A + bM\left(\beta\ (-1+\gamma)^2 + \gamma - 2\,\gamma^2\right)\ n_B\right)\ /\ ((\beta+\gamma)\ (-1+2\,\gamma)\ n_B)\right), 2\right]\right]$$

$$Simplify\left[Tnb\left[(\gamma\ (aM\ (1+\beta)\ \gamma\, n_A + bM\ (\beta\ (-1+\gamma) - \gamma)\ n_B))\ /\ ((\beta+\gamma)\ (-1+2\,\gamma)\ n_A),\ -\left(\left(aM\,\beta\ (-1+\gamma)\ \gamma\, n_A + bM\left(\beta\ (-1+\gamma)^2 + \gamma - 2\,\gamma^2\right)\ n_B\right)\ /\ ((\beta+\gamma)\ (-1+2\,\gamma)\ n_B)\right), \beta, \gamma, 2\right]\right]$$

$$\frac{aM\,\gamma\, n_A + bM\ (-1+\gamma)\ n_B}{-1+2\,\gamma}$$

$$\frac{\gamma^2\ (aM\ (-1+\gamma)\ n_A + bM\,\gamma\, n_B)^2}{2\ (1-2\,\gamma)^2\ (\beta+\gamma)\ n_B}$$

The equilibrium payoffs of the median residents are calculated in the next two lines.

$$\text{uA}\left[\frac{\text{aM}\,\gamma\,n_A+\text{bM}\,(-1+\gamma)\,n_B}{-1+2\,\gamma},\ \text{aM},\ \frac{\gamma^2\,(\text{aM}\,(-1+\gamma)\,n_A+\text{bM}\,\gamma\,n_B)^2}{2\,(1-2\,\gamma)^2\,(\beta+\gamma)\,n_B},\ 2\right]$$

$$\text{uB}\left[\frac{\text{aM}\,\gamma\,n_A+\text{bM}\,(-1+\gamma)\,n_B}{-1+2\,\gamma},\ \text{bM},\ \frac{\gamma^2\,(\text{aM}\,(-1+\gamma)\,n_A+\text{bM}\,\gamma\,n_B)^2}{2\,(1-2\,\gamma)^2\,(\beta+\gamma)\,n_B},\ 2\right]$$

$$\frac{\text{aM}\,(\text{aM}\,\gamma\,n_A+\text{bM}\,(-1+\gamma)\,n_B)}{-1+2\,\gamma}-\frac{\gamma\,(\text{aM}\,\gamma\,n_A+\text{bM}\,(-1+\gamma)\,n_B)^2}{2\,(-1+2\,\gamma)^2\,n_A}+\frac{\gamma^2\,(\text{aM}\,(-1+\gamma)\,n_A+\text{bM}\,\gamma\,n_B)^2}{2\,(1-2\,\gamma)^2\,(\beta+\gamma)\,n_A}$$

$$\frac{\text{bM}\,(\text{aM}\,\gamma\,n_A+\text{bM}\,(-1+\gamma)\,n_B)}{-1+2\,\gamma}-\frac{(1-\gamma)\,(\text{aM}\,\gamma\,n_A+\text{bM}\,(-1+\gamma)\,n_B)^2}{2\,(-1+2\,\gamma)^2\,n_B}-\frac{\gamma^2\,(\text{aM}\,(-1+\gamma)\,n_A+\text{bM}\,\gamma\,n_B)^2}{2\,(1-2\,\gamma)^2\,(\beta+\gamma)\,n_B}$$

Table 1 in Section 4.2 of the main text

We examine what γ maximizes the payoff to region A's median resident. First, we differentiate the payoff to region A's median resident with respect to γ in the next line.

$$\text{uA}\left[\frac{\text{aM}\,\gamma\,n_A+\text{bM}\,(-1+\gamma)\,n_B}{-1+2\,\gamma},\ \text{aM},\ \frac{\gamma^2\,(\text{aM}\,(-1+\gamma)\,n_A+\text{bM}\,\gamma\,n_B)^2}{2\,(1-2\,\gamma)^2\,(\beta+\gamma)\,n_B},\ 2\right]$$

$$\frac{\text{aM}\,(\text{aM}\,\gamma\,n_A+\text{bM}\,(-1+\gamma)\,n_B)}{-1+2\,\gamma}-\frac{\gamma\,(\text{aM}\,\gamma\,n_A+\text{bM}\,(-1+\gamma)\,n_B)^2}{2\,(-1+2\,\gamma)^2\,n_A}+\frac{\gamma^2\,(\text{aM}\,(-1+\gamma)\,n_A+\text{bM}\,\gamma\,n_B)^2}{2\,(1-2\,\gamma)^2\,(\beta+\gamma)\,n_A}$$

The function "MUA" is a reformulation of the above derivative coefficient of the payoff to region A's median resident.

$$\text{MUA}[\text{aM\_},\ \text{bM\_},\ \text{nA\_},\ \text{nB\_},\ \beta\text{\_}] := \frac{1}{2\,(\beta+\gamma)^2\,(-1+2\,\gamma)^3\,\text{nA}}\Big(\text{aM}^2\,\big(\gamma^2-2\,\gamma^3-2\,\beta\,\gamma\,(-1+\gamma+\gamma^2)+\beta^2\,(2-4\,\gamma+3\,\gamma^2-2\,\gamma^3)\big)\,\text{nA}\hat{}2 - 2\,\text{aM}\,\text{bM}\,\big(\beta\,(2-3\,\gamma)\,\gamma+\gamma^2-2\,\gamma^3+\beta^2\,(-1+\gamma)^2\,(1+2\,\gamma)\big)\,\text{nA}\,\text{nB} + \text{bM}^2\,\big(2\,\beta\,(-1+\gamma)^2\,\gamma+\gamma^2-2\,\gamma^3+\beta^2\,(1-2\,\gamma+3\,\gamma^2-2\,\gamma^3)\big)\,\text{nB}\hat{}2\Big)$$

We calculate $\gamma[a_M]$ for all seven cases in Table 1. In this calculation, we assume that $n_B = 1$, $\text{bM} = 0.5$, and $\beta = 0.5$.

Case (1)

Solving the FOC for region A’s median resident yields

$$\text{Reduce}\left[\left\{\text{MUA}\left[1+\frac{\text{nA}}{2},\ 1/2,\ \text{nA},\ 1,\ 1/2\right] == 0,\ 0 \le \gamma \le 1,\ \text{nA} == 1\right\},\ \gamma\right]$$

N[%]

$\text{nA} == 1\ \&\&\ \gamma == \text{Root}\left[-13+22\,\#1-56\,\#1^2+96\,\#1^3\ \&,\ 1\right]$

$\text{nA} == 1.\ \&\&\ \gamma == 0.586363$

The solution satisfies the SOC because

```
D[MUA[1 + nA/2, 1/2, nA, 1, 1/2], γ]  /. nA → 1  /. γ -> 0.5863632562813743`
```

```
-284.397
```

Thus, $\gamma[a_M]$=0.586363 is the optimal γ for aM in Case (1).

Case (2)

Solving the FOC for region A's median resident yields

```
Reduce[{MUA[1 + nA/2, 1/2, nA, 1, 1/2] == 0, 0 ≤ γ ≤ 1, nA == 3/2}, γ]
N[%]
```

```
nA == 3/2 && γ == Root[-730 + 640 #1 - 2147 #1^2 + 5262 #1^3 &, 1]
```

```
nA == 1.5 && γ == 0.595239
```

The solution satisfies the SOC because

```
D[MUA[1 + nA/2, 1/2, nA, 1, 1/2], γ]  /. nA → 3/2  /. γ -> 0.5952389761857527`
```

```
-577.565
```

Thus, $\gamma[a_M]$=0.595239 is the optimal γ for aM in Case (2).

Case (3)

Solving the FOC for region A's median resident yields

```
Reduce[{MUA[1 + nA/2, 1/2, nA, 1, 1/2] == 0, 0 ≤ γ ≤ 1, nA == 2}, γ]
N[%]
```

```
nA == 2 && γ == Root[-113 + 62 #1 - 271 #1^2 + 806 #1^3 &, 1]
```

```
nA == 2. && γ == 0.598785
```

The solution satisfies the SOC because

```
D[MUA[1 + nA/2, 1/2, nA, 1, 1/2], γ]  /. nA → 2  /. γ -> 0.5987851737135482`
```

```
-1014.3
```

Thus, $\gamma[a_M]$=0.598785 is the optimal γ for aM in Case (3).

Case (4)

Solving the FOC for region A's median resident yields

```
Reduce[{MUA[1 + nA/2, 1/2, nA, 1, 1/2] == 0, 0 ≤ γ ≤ 1, nA == 5/2}, γ]
N[%]
```

$nA == \frac{5}{2}$ && γ == Root[− 3706 + 1408 #1 − 7859 #1^2 + 26 286 #1^3 &, 1]

nA == 2.5 && γ == 0.600621

The solution satisfies the SOC because

```
D[MUA[1 + nA/2, 1/2, nA, 1, 1/2], γ] /. nA → 2.5 /. γ -> 0.6006210706462157`
```

−1615.51

Thus, $\gamma[a_M]$=0.600621 is the optimal γ for aM in Case (4).

Case (5)

Solving the FOC for region A’s median resident yields

```
Reduce[{MUA[1 + nA/2, 1/2, nA, 1, 1/2] == 0, 0 ≤ γ ≤ 1, nA == 3}, γ]
N[%]
```

nA == 3 && γ == Root[− 421 + 118 #1 − 824 #1^2 + 2976 #1^3 &, 1]

nA == 3. && γ == 0.601712

The solution satisfies the SOC because

```
D[MUA[1 + nA/2, 1/2, nA, 1, 1/2], γ] /. nA → 3 /. γ -> 0.6017115778932084`
```

−2404.84

Thus, $\gamma[a_M]$=0.601712 is the optimal γ for aM in Case (5).

Case (6)

Solving the FOC for region A’s median resident yields

```
Reduce[{MUA[1 + nA/2, 1/2, nA, 1, 1/2] == 0, 0 ≤ γ ≤ 1, nA == 7/2}, γ]
N[%]
```

$nA == \frac{7}{2}$ && γ == Root[− 11 258 + 2432 #1 − 20 851 #1^2 + 79 406 #1^3 &, 1]

nA == 3.5 && γ == 0.602418

The solution satisfies the SOC because

```
D[MUA[1 + nA/2, 1/2, nA, 1, 1/2], γ] /. nA → 3.5 /. γ -> 0.6024180908308389`
```

−3406.47

Thus, $\gamma[a_M]$=0.602418 is the optimal γ for aM in Case (6).

Case (7)

Solving the FOC for region A's median resident yields

```
Reduce[{MUA[1 + nA/2, 1 / 2, nA, 1, 1 / 2] == 0, 0 ≤ γ ≤ 1, nA == 4}, γ]
N[%]
```

nA == 4 && γ == Root[-1105 + 190 #1 - 1967 #1^2 + 7782 #1^3 &, 1]

nA == 4. && γ == 0.602904

The solution satisfies the SOC because

```
D[MUA[1 + nA/2, 1 / 2, nA, 1, 1 / 2], γ] /. nA → 4 /. γ -> 0.6029043597227752`
```

-4644.72

Thus, $\gamma[a_M]$=0.602904 is the optimal γ for aM in Case (7).

Table 2 in Section 4.3 of the main text

The payoff to a resindet in the regions are defined as follows in this section. The first (second) line is the payoff to a region A's (regino B's, respectively) resident:

```
UA[x_, a_, T_, α_, IA_, nA_, nB_] := a x - γ/(nA * α) * x^α + nB/nA * T + IA/nA + t a
UB[x_, b_, T_, α_, IB_, nA_, nB_] := b x - (1 - γ)/(nB * α) * x^α - T + IB/nB + t (N - a)
```

We consider the case of α=2. In this case, x^{nb} ("xnb" in the next line) and T^{nb} ("tnb" in the next line) are calculated as follows:

$$\text{xnb} = \frac{\text{aM}\,\gamma\,\text{nA} + \text{bM}\,(-1+\gamma)\,\text{nB}}{-1+2\gamma}$$

$$\text{tnb} = \frac{\gamma^2\,(\text{aM}\,(-1+\gamma)\,\text{nA} + \text{bM}\,\gamma\,\text{nB})^2}{2\,(1-2\gamma)^2\,(\beta+\gamma)\,\text{nB}}$$

$$\frac{\text{bM nB}\,(-1+\gamma) + \text{aM nA}\,\gamma}{-1+2\gamma}$$

$$\frac{\gamma^2\,(\text{aM nA}\,(-1+\gamma) + \text{bM nB}\,\gamma)^2}{2\,\text{nB}\,(1-2\gamma)^2\,(\beta+\gamma)}$$

$$\frac{\gamma^2\,(\text{aM nA}\,(-1+\gamma) + \text{bM nB}\,\gamma)^2}{2\,\text{nB}\,(1-2\gamma)^2\,(\beta+\gamma)}$$

$$\frac{\text{bM nB}\,(-1+\gamma) + \text{aM nA}\,\gamma}{-1+2\gamma}$$

$$\frac{\gamma^2\,(\text{aM nA}\,(-1+\gamma) + \text{bM nB}\,\gamma)^2}{2\,\text{nB}\,(1-2\gamma)^2\,(\beta+\gamma)}$$

$$\frac{\text{bM nB}\,(-1+\gamma) + \text{aM nA}\,\gamma}{-1+2\gamma}$$

If α=2, the payoffs are recalculated as follows:

```
UA[xnb, a, tnb, 2, IA, nA, nB]
UB[xnb, a, tnb, 2, IB, nA, nB]
```

$$\frac{\text{IA}}{\text{nA}} + \text{a t} + \frac{\text{a}\,(\text{bM nB}\,(-1+\gamma) + \text{aM nA}\,\gamma)}{-1+2\gamma} - \frac{\gamma\,(\text{bM nB}\,(-1+\gamma) + \text{aM nA}\,\gamma)^2}{2\,\text{nA}\,(-1+2\gamma)^2} + \frac{\gamma^2\,(\text{aM nA}\,(-1+\gamma) + \text{bM nB}\,\gamma)^2}{2\,\text{nA}\,(1-2\gamma)^2\,(\beta+\gamma)}$$

$$\frac{\text{IB}}{\text{nB}} + (-\text{a}+\text{N})\,\text{t} + \frac{\text{a}\,(\text{bM nB}\,(-1+\gamma) + \text{aM nA}\,\gamma)}{-1+2\gamma} - \frac{(1-\gamma)\,(\text{bM nB}\,(-1+\gamma) + \text{aM nA}\,\gamma)^2}{2\,\text{nB}\,(-1+2\gamma)^2} - \frac{\gamma^2\,(\text{aM nA}\,(-1+\gamma) + \text{bM nB}\,\gamma)^2}{2\,\text{nB}\,(1-2\gamma)^2\,(\beta+\gamma)}$$

The following line calculates the migration equilibrium condition: that is, "a" in the next line is the individual that is indifferent between residing in region A and region B.

```
Solve[UA[xnb, a, tnb, 2, IA, nA, nB] == UB[xnb, a, tnb, 2, IB, nA, nB], a]
```

$$\left\{\left\{\text{a} \to \frac{1}{2\,\text{t}}\left(-\frac{\text{IA}}{\text{nA}} + \frac{\text{IB}}{\text{nB}} + \text{N t} - \frac{(1-\gamma)\,(\text{bM nB}\,(-1+\gamma) + \text{aM nA}\,\gamma)^2}{2\,\text{nB}\,(-1+2\gamma)^2} + \frac{\gamma\,(\text{bM nB}\,(-1+\gamma) + \text{aM nA}\,\gamma)^2}{2\,\text{nA}\,(-1+2\gamma)^2} - \frac{\gamma^2\,(\text{aM nA}\,(-1+\gamma) + \text{bM nB}\,\gamma)^2}{2\,\text{nA}\,(1-2\gamma)^2\,(\beta+\gamma)} - \frac{\gamma^2\,(\text{aM nA}\,(-1+\gamma) + \text{bM nB}\,\gamma)^2}{2\,\text{nB}\,(1-2\gamma)^2\,(\beta+\gamma)}\right)\right\}\right\}$$

The following calculation is done based on the assumption that $\beta = 1/2$ and $I_A = I_B$(the same income across the regions).

The following function "Li" calculates the Lindahl price:

$$\text{Li}[\text{aM\_}, \text{nA\_}, \text{bM\_}, \text{nB\_}] := \frac{\text{aM} * \text{nA}}{\text{aM} * \text{nA} + \text{bM} * \text{nB}}$$

For each population distribution listed in Table 2, we derive the value of "γ" that stabilizes the population distribution. The calculation is done under the assumption that $n_B = 1$, bM=1/2, $\beta = 1/2$, t=1, and $I_A = I_B = 10$. Note that *under all population distributions in Table 2, the marginal individual is equal to 1.*

Case (1) nA=1, aM=1.5.

In this case, the Lindahl price is as follows:

```
Li[1.5, 1, 1 / 2, 1]
0.75
```

In this case, the marginal individual that is indifferent between residing in region A and region B is calculated as follows:

$$\frac{1}{2\,t}\left(-\frac{IA}{nA}+\frac{IB}{nB}+N\,t-\left((1-\gamma)\,(bM\,nB\,(-1+\gamma)+aM\,nA\,\gamma)^2\right)\Big/\left(2\,nB\,(-1+2\gamma)^2\right)+\frac{\gamma\,(bM\,nB\,(-1+\gamma)+aM\,nA\,\gamma)^2}{2\,nA\,(-1+2\gamma)^2}-\left(\gamma^2\,(aM\,nA\,(-1+\gamma)+bM\,nB\,\gamma)^2\right)\Big/\left(2\,nA\,(1-2\gamma)^2\,(\beta+\gamma)\right)-\left(\gamma^2\,(aM\,nA\,(-1+\gamma)+bM\,nB\,\gamma)^2\right)\Big/\left(2\,nB\,(1-2\gamma)^2\,(\beta+\gamma)\right)\right)\ /.\ nA\to 1\ /.\ nB\to 1\ /.\ bM\to 1/2\ /.\ aM\to 3/2\ /.\ \beta\to 1/2\ /.\ IA\to 10\ /.\ IB\to 10\ /.\ N\to 2$$

$$\frac{1}{2\,t}\left(2\,t-\frac{\left(\frac{3}{2}\,(-1+\gamma)+\frac{\gamma}{2}\right)^2\gamma^2}{(1-2\gamma)^2\left(\frac{1}{2}+\gamma\right)}-\frac{(1-\gamma)\left(\frac{1}{2}\,(-1+\gamma)+\frac{3\gamma}{2}\right)^2}{2\,(-1+2\gamma)^2}+\frac{\gamma\left(\frac{1}{2}\,(-1+\gamma)+\frac{3\gamma}{2}\right)^2}{2\,(-1+2\gamma)^2}\right)$$

In the next line, we calcualte the value of γ at which the marginal individual is equal to 1.

$$\text{Reduce}\left[\left\{\frac{1}{2\,t}\left(2\,t-\frac{\left(\frac{3}{2}\,(-1+\gamma)+\frac{\gamma}{2}\right)^2\gamma^2}{(1-2\gamma)^2\left(\frac{1}{2}+\gamma\right)}-\frac{(1-\gamma)\left(\frac{1}{2}\,(-1+\gamma)+\frac{3\gamma}{2}\right)^2}{2\,(-1+2\gamma)^2}+\frac{\gamma\left(\frac{1}{2}\,(-1+\gamma)+\frac{3\gamma}{2}\right)^2}{2\,(-1+2\gamma)^2}\right)==1,\ \frac{1}{\sqrt{3}}<\gamma\le 1,\ t>0\right\},\ \gamma\right]$$

```
N[
 %]
```

$t > 0\ \&\&\ \gamma == \text{Root}\left[-1+8\,\#1-48\,\#1^2+64\,\#1^3\ \&,\ 1\right]$

$t > 0.\ \&\&\ \gamma == 0.581179$

We find that γ^S=0.581179 in this case.

Case (2) nA=1.5, aM=1.75

In this case, the Lindahl price is as follows:

```
Li[1.75, 1.5, 1/2, 1]
```

```
0.84
```

In this case, the marginal individual that is indifferent between residing in region A and region B is calculated as follows:

$$\frac{1}{2t}\left(-\frac{IA}{nA}+\frac{IB}{nB}+N\,t-\left((1-\gamma)\,(bM\,nB\,(-1+\gamma)+aM\,nA\,\gamma)^2\right)\Big/\left(2\,nB\,(-1+2\gamma)^2\right)+\frac{\gamma\,(bM\,nB\,(-1+\gamma)+aM\,nA\,\gamma)^2}{2\,nA\,(-1+2\gamma)^2}-\left(\gamma^2\,(aM\,nA\,(-1+\gamma)+bM\,nB\,\gamma)^2\right)\Big/\left(2\,nA\,(1-2\gamma)^2\,(\beta+\gamma)\right)-\left(\gamma^2\,(aM\,nA\,(-1+\gamma)+bM\,nB\,\gamma)^2\right)\Big/\left(2\,nB\,(1-2\gamma)^2\,(\beta+\gamma)\right)\right)\;/.\;nA\to 1.5\;/.\;nB\to 1\;/.$$

$$bM\to 1/2\;/.\;aM\to 1.75\;/.\;\beta\to 1/2\;/.\;IA\to 10\;/.\;IB\to 10\;/.\;N\to 2.5$$

$$\frac{1}{2t}\left(3.33333+2.5\,t-\frac{0.833333\left(2.625\,(-1+\gamma)+\frac{\gamma}{2}\right)^2\gamma^2}{(1-2\gamma)^2\left(\frac{1}{2}+\gamma\right)}-\frac{(1-\gamma)\left(\frac{1}{2}(-1+\gamma)+2.625\,\gamma\right)^2}{2\,(-1+2\gamma)^2}+\frac{0.333333\,\gamma\left(\frac{1}{2}(-1+\gamma)+2.625\,\gamma\right)^2}{(-1+2\gamma)^2}\right)$$

In the next line, we calcualte the value of γ at which the marginal individual is equal to 1.

$$\text{Reduce}\left[\left\{\frac{1}{2t}\left(3.3333333333333334`+2.5`\,t-\left(0.8333333333333333`\left(2.625`\,(-1+\gamma)+\frac{\gamma}{2}\right)^2\gamma^2\right)\Big/\left((1-2\gamma)^2\left(\frac{1}{2}+\gamma\right)\right)-\frac{(1-\gamma)\left(\frac{1}{2}(-1+\gamma)+2.625`\,\gamma\right)^2}{2\,(-1+2\gamma)^2}+\left(0.3333333333333333`\,\gamma\left(\frac{1}{2}(-1+\gamma)+2.625`\,\gamma\right)^2\right)\Big/(-1+2\gamma)^2\right)==1,\;\frac{1}{\sqrt{3}}<\gamma\le 1,\;t==1\right\},\gamma\right]$$

$t == 1.\;\&\&\;\gamma == 0.600014$

We find that γ^S=0.600014 in this case.

Case (3) nA=2, aM=2

In this case, the Lindahl price is as follows:

```
N[Li[2, 2, 1 / 2, 1]]
```

0.888889

In this case, the marginal individual that is indifferent between residing in region A and region B is calculated as follows:

$$\frac{1}{2\,t}\left(-\frac{IA}{nA}+\frac{IB}{nB}+N\,t-\left((1-\gamma)\ (bM\,nB\ (-1+\gamma)+aM\,nA\,\gamma)^2\right)\Big/\left(2\,nB\ (-1+2\,\gamma)^2\right)+\frac{\gamma\ (bM\,nB\ (-1+\gamma)+aM\,nA\,\gamma)^2}{2\,nA\ (-1+2\,\gamma)^2}-\left(\gamma^2\ (aM\,nA\ (-1+\gamma)+bM\,nB\,\gamma)^2\right)\Big/\left(2\,nA\ (1-2\,\gamma)^2\ (\beta+\gamma)\right)-\left(\gamma^2\ (aM\,nA\ (-1+\gamma)+bM\,nB\,\gamma)^2\right)\Big/\left(2\,nB\ (1-2\,\gamma)^2\ (\beta+\gamma)\right)\right)\ /.\ nA\to 2\ /.\ nB\to 1\ /.\ bM\to 1/2\ /.\ aM\to 2\ /.\ \beta\to 1/2\ /.\ IA\to 10\ /.\ IB\to 10\ /.\ N\to 3$$

$$\frac{1}{2\,t}\left(5+3\,t-\frac{3\left(4\ (-1+\gamma)+\frac{\gamma}{2}\right)^2\gamma^2}{4\ (1-2\,\gamma)^2\left(\frac{1}{2}+\gamma\right)}-\frac{(1-\gamma)\left(\frac{1}{2}\ (-1+\gamma)+4\,\gamma\right)^2}{2\ (-1+2\,\gamma)^2}+\frac{\gamma\left(\frac{1}{2}\ (-1+\gamma)+4\,\gamma\right)^2}{4\ (-1+2\,\gamma)^2}\right)$$

In the next line, we calcualte the value of γ at which the marginal individual is equal to 1.

$$\text{Reduce}\left[\left\{\frac{1}{2\,t}\left(5+3\,t-\frac{3\left(4\ (-1+\gamma)+\frac{\gamma}{2}\right)^2\gamma^2}{4\ (1-2\,\gamma)^2\left(\frac{1}{2}+\gamma\right)}-\frac{(1-\gamma)\left(\frac{1}{2}\ (-1+\gamma)+4\,\gamma\right)^2}{2\ (-1+2\,\gamma)^2}+\frac{\gamma\left(\frac{1}{2}\ (-1+\gamma)+4\,\gamma\right)^2}{4\ (-1+2\,\gamma)^2}\right)==1,\ \frac{1}{\sqrt{3}}<\gamma\le 1,\ t==1\right\},\ \gamma\right]$$

```
N[
 %]
```

$t == 1\ \&\&\ \gamma == \text{Root}\left[94-157\,\#1-906\,\#1^2+1443\,\#1^3\ \&,\ 3\right]$

$t == 1.\ \&\&\ \gamma == 0.638465$

We find that γ^S=0.638465 in this case.

Case (4) nA=2.5, aM=2.25

In this case, the Lindahl price is as follows:

```
N[Li[2.25, 2.5, 1 / 2, 1]]
```

0.918367

In this case, the marginal individual that is indifferent between residing in region A and region B is calculated as follows:

```
1/(2 t) (-IA/nA + IB/nB + N t - ((1 - γ) (bM nB (-1 + γ) + aM nA γ)^2) / (2 nB (-1 + 2 γ)^2) +
    γ (bM nB (-1 + γ) + aM nA γ)^2 / (2 nA (-1 + 2 γ)^2) - (γ^2 (aM nA (-1 + γ) + bM nB γ)^2) /
    (2 nA (1 - 2 γ)^2 (β + γ)) - (γ^2 (aM nA (-1 + γ) + bM nB γ)^2) /
    (2 nB (1 - 2 γ)^2 (β + γ))) /. nA → 2.5 /. nB → 1 /.
  bM → 1/2 /. aM → 2.25 /. β → 1/2 /. IA → 10 /. IB → 10 /. N → 3.5
```

$$\frac{1}{2\,t}\left(6.+3.5\,t-\frac{0.7\left(5.625\,(-1+\gamma)+\frac{\gamma}{2}\right)^2\gamma^2}{(1-2\,\gamma)^2\left(\frac{1}{2}+\gamma\right)}-\frac{(1-\gamma)\left(\frac{1}{2}\,(-1+\gamma)+5.625\,\gamma\right)^2}{2\,(-1+2\,\gamma)^2}+\frac{0.2\,\gamma\left(\frac{1}{2}\,(-1+\gamma)+5.625\,\gamma\right)^2}{(-1+2\,\gamma)^2}\right)$$

In the next line, we calcualte the value of γ at which the marginal individual is equal to 1.

```
Reduce[{1/(2 t) (6.` + 3.5` t - 0.7` (5.625` (-1 + γ) + γ/2)^2 γ^2 / ((1 - 2 γ)^2 (1/2 + γ)) - (1 - γ) (1/2 (-1 + γ) + 5.625` γ)^2 / (2 (-1 + 2 γ)^2) +
    0.2` γ (1/2 (-1 + γ) + 5.625` γ)^2 / (-1 + 2 γ)^2) == 1, 1/Sqrt[3] < γ ≤ 1, t == 1}, γ]

N[
 %]
```

$t == 1.\ \&\&\ \gamma == 0.677132$

$t == 1.\ \&\&\ \gamma == 0.677132$

We find that γ^S=0.677132 in this case.

Case (5) nA=3, aM=2.5

In this case, the Lindahl price is as follows:

```
N[Li[2.5, 3, 1/2, 1]]
```

0.9375

In this case, the marginal individual that is indifferent between residing in region A and region B is calculated as follows:

$$\frac{1}{2\,t}\left(-\frac{IA}{nA}+\frac{IB}{nB}+N\,t-\left((1-\gamma)\,(bM\,nB\,(-1+\gamma)+aM\,nA\,\gamma)^2\right)\Big/\left(2\,nB\,(-1+2\,\gamma)^2\right)+\frac{\gamma\,(bM\,nB\,(-1+\gamma)+aM\,nA\,\gamma)^2}{2\,nA\,(-1+2\,\gamma)^2}-\left(\gamma^2\,(aM\,nA\,(-1+\gamma)+bM\,nB\,\gamma)^2\right)\Big/\left(2\,nA\,(1-2\,\gamma)^2\,(\beta+\gamma)\right)-\left(\gamma^2\,(aM\,nA\,(-1+\gamma)+bM\,nB\,\gamma)^2\right)\Big/\left(2\,nB\,(1-2\,\gamma)^2\,(\beta+\gamma)\right)\right)\text{ /. }nA\to 3\text{ /. }nB\to 1\text{ /. }bM\to 1/2\text{ /. }aM\to 2.5\text{ /. }\beta\to 1/2\text{ /. }IA\to 10\text{ /. }IB\to 10\text{ /. }N\to 4$$

$$\frac{1}{2\,t}\left(\frac{20}{3}+4\,t-\frac{2\left(7.5\,(-1+\gamma)+\frac{\gamma}{2}\right)^2\gamma^2}{3\,(1-2\,\gamma)^2\left(\frac{1}{2}+\gamma\right)}-\frac{(1-\gamma)\left(\frac{1}{2}\,(-1+\gamma)+7.5\,\gamma\right)^2}{2\,(-1+2\,\gamma)^2}+\frac{\gamma\left(\frac{1}{2}\,(-1+\gamma)+7.5\,\gamma\right)^2}{6\,(-1+2\,\gamma)^2}\right)$$

In the next line, we calcualte the value of γ at which the marginal individual is equal to 1.

$$\text{Reduce}\left[\left\{\frac{1}{2\,t}\left(\frac{20}{3}+4\,t-\frac{2\left(7.5`\,(-1+\gamma)+\frac{\gamma}{2}\right)^2\gamma^2}{3\,(1-2\,\gamma)^2\left(\frac{1}{2}+\gamma\right)}-\frac{(1-\gamma)\left(\frac{1}{2}\,(-1+\gamma)+7.5`\,\gamma\right)^2}{2\,(-1+2\,\gamma)^2}+\frac{\gamma\left(\frac{1}{2}\,(-1+\gamma)+7.5`\,\gamma\right)^2}{6\,(-1+2\,\gamma)^2}\right)==1,\ \frac{1}{\sqrt{3}}<\gamma\le 1,\ t==1\right\},\ \gamma\right]$$

```
N[
 %]
```

t == 1. && γ == 0.712904

t == 1. && γ == 0.712904

We find that γ^S=0.712904 in this case.

Case (6) nA=3.5, aM=2.75

In this case, the Lindahl price is as follows:

```
N[Li[2.75, 3.5, 1 / 2, 1]]
```

0.950617

In this case, the marginal individual that is indifferent between residing in region A and region B is calculated as follows:

```
1/(2 t) (-IA/nA + IB/nB + N t - ((1 - γ) (bM nB (-1 + γ) + aM nA γ)^2) / (2 nB (-1 + 2 γ)^2) +
    γ (bM nB (-1 + γ) + aM nA γ)^2 / (2 nA (-1 + 2 γ)^2) - (γ^2 (aM nA (-1 + γ) + bM nB γ)^2) /
    (2 nA (1 - 2 γ)^2 (β + γ)) - (γ^2 (aM nA (-1 + γ) + bM nB γ)^2) /
    (2 nB (1 - 2 γ)^2 (β + γ)))  /. nA → 3.5 /. nB → 1 /.
  bM → 1 / 2 /. aM → 2.75 /. β → 1 / 2 /. IA → 10 /. IB → 10 /. N → 4.5
```

$$\frac{1}{2t}\left(7.14286+4.5\,t-\frac{0.642857\left(9.625\,(-1+\gamma)+\frac{\gamma}{2}\right)^2\gamma^2}{(1-2\gamma)^2\left(\frac{1}{2}+\gamma\right)}-\frac{(1-\gamma)\left(\frac{1}{2}(-1+\gamma)+9.625\,\gamma\right)^2}{2\,(-1+2\gamma)^2}+\frac{0.142857\,\gamma\left(\frac{1}{2}(-1+\gamma)+9.625\,\gamma\right)^2}{(-1+2\gamma)^2}\right)$$

In the next line, we calcualte the value of γ at which the marginal individual is equal to 1.

```
Reduce[{1/(2 t) (7.142857142857143` + 4.5` t -
     (0.6428571428571428` (9.625` (-1 + γ) + γ/2)^2 γ^2) / ((1 - 2 γ)^2 (1/2 + γ)) -
     (1 - γ) (1/2 (-1 + γ) + 9.625` γ)^2 / (2 (-1 + 2 γ)^2) + (0.14285714285714285` γ (1/2 (-1 + γ) + 9.625` γ)^2) /
     (-1 + 2 γ)^2) == 1, 1/Sqrt[3] < γ ≤ 1, t == 1}, γ]
N[
 %]
```

```
t == 1. && γ == 0.744515
```

```
t == 1. && γ == 0.744515
```

We find that γ^S=0.744515 in this case.

Case (7) nA=4, aM=3

In this case, the Lindahl price is as follows:

```
N[Li[3, 4, 1 / 2, 1]]
```

```
0.96
```

In this case, the marginal individual that is indifferent between residing in region A and region B is calculated as follows:

```
1/(2 t) (-IA/nA + IB/nB + N t - ((1 - γ) (bM nB (-1 + γ) + aM nA γ)^2) /
      (2 nB (-1 + 2 γ)^2) + (γ (bM nB (-1 + γ) + aM nA γ)^2)/(2 nA (-1 + 2 γ)^2) -
    (γ^2 (aM nA (-1 + γ) + bM nB γ)^2) / (2 nA (1 - 2 γ)^2 (β + γ)) -
    (γ^2 (aM nA (-1 + γ) + bM nB γ)^2) / (2 nB (1 - 2 γ)^2 (β + γ))) /. nA → 4 /. nB → 1 /.
  bM → 1 / 2 /. aM → 3 /. β → 1 / 2 /. IA → 10 /. IB → 10 /. N → 5
```

$$\frac{1}{2\,t}\left(\frac{15}{2}+5\,t-\frac{5\left(12\,(-1+\gamma)+\frac{\gamma}{2}\right)^2\gamma^2}{8\,(1-2\,\gamma)^2\left(\frac{1}{2}+\gamma\right)}-\frac{(1-\gamma)\left(\frac{1}{2}\,(-1+\gamma)+12\,\gamma\right)^2}{2\,(-1+2\,\gamma)^2}+\frac{\gamma\left(\frac{1}{2}\,(-1+\gamma)+12\,\gamma\right)^2}{8\,(-1+2\,\gamma)^2}\right)$$

In the next line, we calcualte the value of γ at which the marginal individual is equal to 1.

```
Reduce[{1/(2 t)
    (15/2 + 5 t - (5 (12 (-1 + γ) + γ/2)^2 γ^2)/(8 (1 - 2 γ)^2 (1/2 + γ)) - ((1 - γ) (1/2 (-1 + γ) + 12 γ)^2)/(2 (-1 + 2 γ)^2) + (γ (1/2 (-1 + γ) + 12 γ)^2)/(8 (-1 + 2 γ)^2)) ==
   1, 1/√3 < γ ≤ 1, t == 1}, γ]
N[
 %]
```

$t == 1 \,\&\&\, \gamma == \text{Root}\left[332-475\,\#1-9444\,\#1^2+12\,313\,\#1^3\,\&,\,3\right]$

$t == 1.\,\&\&\, \gamma == 0.771707$

We find that γ^S=0.771707 in this case.

Table 3 in Section A of Online Appendix

In this section, we consider the case of α=3. x^{nb} and T^{nb} are calculated as follows:

```
Xnb[aR, bR, 3]
Simplify[Tnb[aR, bR, β, γ, 3]]
```

$$\sqrt{aR\,n_A+bR\,n_B}$$

$$\frac{1}{3\,n_B}\left(\beta\left(\frac{aR\,n_A}{\gamma}\right)^{3/2}+2\,\gamma\left(\frac{aR\,n_A}{\gamma}\right)^{3/2}-3\,\beta\,\gamma\left(\frac{aR\,n_A}{\gamma}\right)^{3/2}+3\,aR\,(-1+\beta)\,n_A\sqrt{aR\,n_A+bR\,n_B}-\beta\,(aR\,n_A+bR\,n_B)^{3/2}+\gamma\,(aR\,n_A+bR\,n_B)^{3/2}-3\,bR\,\beta\,n_B\left(\sqrt{\frac{aR\,n_A}{\gamma}}-\sqrt{aR\,n_A+bR\,n_B}\right)\right)$$

The payoff function of region A's median resident *aM*, U(x^{nb},T^{nb}; aM), is calculated in the next line:

```
uA[Xnb[aR, bR, 3], aM, Tnb[aR, bR, β, γ, 3], 3]
```

$$aM\sqrt{aR\, n_A + bR\, n_B} - \frac{\gamma\,(aR\, n_A + bR\, n_B)^{3/2}}{3\, n_A} + \frac{1}{3\, n_A}\left(\beta\left(\frac{aR\, n_A}{\gamma}\right)^{3/2} + 2\,\gamma\left(\frac{aR\, n_A}{\gamma}\right)^{3/2} - 3\,\beta\,\gamma\left(\frac{aR\, n_A}{\gamma}\right)^{3/2} + 3\, aR\,(-1+\beta)\, n_A\sqrt{aR\, n_A + bR\, n_B} - \beta\,(aR\, n_A + bR\, n_B)^{3/2} + \gamma\,(aR\, n_A + bR\, n_B)^{3/2} - 3\, bR\,\beta\, n_B\left(\sqrt{\frac{aR\, n_A}{\gamma}} - \sqrt{aR\, n_A + bR\, n_B}\right)\right)$$

The first derivative of U(x^{nb},T^{nb}; aM) with respect to aR is calculated in the next line:

```
D[uA[Xnb[aR, bR, 3], aM, Tnb[aR, bR, β, γ, 3], 3], aR]
```

$$\frac{aM\, n_A}{2\sqrt{aR\, n_A + bR\, n_B}} - \frac{1}{2}\,\gamma\sqrt{aR\, n_A + bR\, n_B} + \frac{1}{3\, n_A}\left(3\, n_A\sqrt{\frac{aR\, n_A}{\gamma}} - \frac{9}{2}\,\beta\, n_A\sqrt{\frac{aR\, n_A}{\gamma}} + \frac{3\,\beta\, n_A\sqrt{\frac{aR\, n_A}{\gamma}}}{2\,\gamma} + \frac{3\, aR\,(-1+\beta)\, n_A^2}{2\sqrt{aR\, n_A + bR\, n_B}} + 3\,(-1+\beta)\, n_A\sqrt{aR\, n_A + bR\, n_B} - \frac{3}{2}\,\beta\, n_A\sqrt{aR\, n_A + bR\, n_B} + \frac{3}{2}\,\gamma\, n_A\sqrt{aR\, n_A + bR\, n_B} - 3\, bR\,\beta\, n_B\left(\frac{n_A}{2\,\gamma\sqrt{\frac{aR\, n_A}{\gamma}}} - \frac{n_A}{2\sqrt{aR\, n_A + bR\, n_B}}\right)\right)$$

In the next line, the second derivative of U(x^{nb},T^{nb}; aM) with respect ot aR is calculated and, then, it is evaluated at aR=aM and bR=bM.

```
D[D[uA[Xnb[aR, bR, 3], aM, Tnb[aR, bR, β, γ, 3], 3], aR], aR]
Simplify[% /. aR → aM /. bR → bM]
```

$$-\frac{aM\, n_A^2}{4\,(aR\, n_A + bR\, n_B)^{3/2}} - \frac{\gamma\, n_A}{4\sqrt{aR\, n_A + bR\, n_B}} + \frac{1}{3\, n_A}\left(\frac{3\,\beta\, n_A^2}{4\,\gamma^2\sqrt{\frac{aR\, n_A}{\gamma}}} + \frac{3\, n_A^2}{2\,\gamma\sqrt{\frac{aR\, n_A}{\gamma}}} - \frac{9\,\beta\, n_A^2}{4\,\gamma\sqrt{\frac{aR\, n_A}{\gamma}}} - \frac{3\, aR\,(-1+\beta)\, n_A^3}{4\,(aR\, n_A + bR\, n_B)^{3/2}} + \frac{3\,(-1+\beta)\, n_A^2}{\sqrt{aR\, n_A + bR\, n_B}} - \frac{3\,\beta\, n_A^2}{4\sqrt{aR\, n_A + bR\, n_B}} + \frac{3\,\gamma\, n_A^2}{4\sqrt{aR\, n_A + bR\, n_B}} - 3\, bR\,\beta\, n_B\left(-\frac{n_A^2}{4\,\gamma^2\left(\frac{aR\, n_A}{\gamma}\right)^{3/2}} + \frac{n_A^2}{4\,(aR\, n_A + bR\, n_B)^{3/2}}\right)\right)$$

$$\left(n_A\left(bM\,\beta\,\gamma\, n_B\sqrt{aM\, n_A + bM\, n_B} + aM\, n_A\left(2\,(-2+\beta)\,\gamma^2\sqrt{\frac{aM\, n_A}{\gamma}} + \beta\sqrt{aM\, n_A + bM\, n_B} + (2-3\,\beta)\,\gamma\sqrt{aM\, n_A + bM\, n_B}\right)\right)\right)\Big/\left(4\,\gamma^3\left(\frac{aM\, n_A}{\gamma}\right)^{3/2}\sqrt{aM\, n_A + bM\, n_B}\right)$$

Rearranging the second derivative of U(x^{nb},T^{nb}; aM) with respect ot aR yields the formula in the next line:

Factor$\Big[$

$$\left(n_A\left(bM\,\beta\,\gamma\, n_B\sqrt{aM\, n_A + bM\, n_B} + aM\, n_A\left(2\,(-2+\beta)\,\gamma^2\sqrt{\frac{aM\, n_A}{\gamma}} + \beta\sqrt{aM\, n_A + bM\, n_B} + (2-3\,\beta)\,\gamma\sqrt{aM\, n_A + bM\, n_B}\right)\right)\right)\Big/\left(4\,\gamma^3\left(\frac{aM\, n_A}{\gamma}\right)^{3/2}\sqrt{aM\, n_A + bM\, n_B}\right)\Big]$$

$$\left(n_A\left(-4\, aM\,\gamma^2\, n_A\sqrt{\frac{aM\, n_A}{\gamma}} + 2\, aM\,\beta\,\gamma^2\, n_A\sqrt{\frac{aM\, n_A}{\gamma}} + aM\,\beta\, n_A\sqrt{aM\, n_A + bM\, n_B} + 2\, aM\,\gamma\, n_A\sqrt{aM\, n_A + bM\, n_B} - 3\, aM\,\beta\,\gamma\, n_A\sqrt{aM\, n_A + bM\, n_B} + bM\,\beta\,\gamma\, n_B\sqrt{aM\, n_A + bM\, n_B}\right)\right)\Big/\left(4\,\gamma^3\left(\frac{aM\, n_A}{\gamma}\right)^{3/2}\sqrt{aM\, n_A + bM\, n_B}\right)$$

For each case of Cases (1)-(7) in Table 3, we calculate under which condition of γ the (local) second-order condition of region A's median resident is satisfied. In this calculation, we assume that β=1/2, n_B = 1, and bM=1/2.

Case (1)

$$\text{Reduce}\left[\frac{1}{4\gamma^3\left(\frac{aM\,n_A}{\gamma}\right)^{3/2}\sqrt{aM\,n_A+bM\,n_B}}n_A\left(-4\,aM\,\gamma^2\,n_A\sqrt{\frac{aM\,n_A}{\gamma}}+2\,aM\,\beta\,\gamma^2\,n_A\sqrt{\frac{aM\,n_A}{\gamma}}+aM\,\beta\,n_A\sqrt{aM\,n_A+bM\,n_B}+2\,aM\,\gamma\,n_A\sqrt{aM\,n_A+bM\,n_B}-3\,aM\,\beta\,\gamma\,n_A\sqrt{aM\,n_A+bM\,n_B}+bM\,\beta\,\gamma\,n_B\sqrt{aM\,n_A+bM\,n_B}\right)<0\,\&\&\,\beta==0.5\,\&\&\,1\geq\gamma>0\,\&\&\,aM==1.5\,\&\&\,n_A==1.0\,\&\&\,bM==0.5\,\&\&\,n_B==1.0,\ \gamma,\ \text{Reals}\right]$$

$n_A == 1.\ \&\&\ n_B == 1.\ \&\&\ \beta == 0.5\ \&\&\ bM == 0.5\ \&\&\ aM == 1.5\ \&\&\ 0.458024 < \gamma \leq 1.$

Thus, the second-order condition for regino A's median resident holds if and only if $0.458024 < \gamma \leq 1$.

Case (2)

$$\text{Reduce}\left[\frac{1}{4\gamma^3\left(\frac{aM\,n_A}{\gamma}\right)^{3/2}\sqrt{aM\,n_A+bM\,n_B}}n_A\left(-4\,aM\,\gamma^2\,n_A\sqrt{\frac{aM\,n_A}{\gamma}}+2\,aM\,\beta\,\gamma^2\,n_A\sqrt{\frac{aM\,n_A}{\gamma}}+aM\,\beta\,n_A\sqrt{aM\,n_A+bM\,n_B}+2\,aM\,\gamma\,n_A\sqrt{aM\,n_A+bM\,n_B}-3\,aM\,\beta\,\gamma\,n_A\sqrt{aM\,n_A+bM\,n_B}+bM\,\beta\,\gamma\,n_B\sqrt{aM\,n_A+bM\,n_B}\right)<0\,\&\&\,\beta==0.5\,\&\&\,1\geq\gamma>0\,\&\&\,aM==1.75\,\&\&\,n_A==1.5\,\&\&\,bM==0.5\,\&\&\,n_B==1.0,\ \gamma,\ \text{Reals}\right]$$

$n_A == 1.5\ \&\&\ n_B == 1.\ \&\&\ \beta == 0.5\ \&\&\ bM == 0.5\ \&\&\ aM == 1.75\ \&\&\ 0.420769 < \gamma \leq 1.$

Thus, the second-order condition for regino A's median resident holds if and only if $0.420769 < \gamma \leq 1$.

Case (3)

$$\text{Reduce}\left[\frac{1}{4\gamma^3\left(\frac{aM\,n_A}{\gamma}\right)^{3/2}\sqrt{aM\,n_A+bM\,n_B}}n_A\left(-4\,aM\,\gamma^2\,n_A\sqrt{\frac{aM\,n_A}{\gamma}}+2\,aM\,\beta\,\gamma^2\,n_A\sqrt{\frac{aM\,n_A}{\gamma}}+aM\,\beta\,n_A\sqrt{aM\,n_A+bM\,n_B}+2\,aM\,\gamma\,n_A\sqrt{aM\,n_A+bM\,n_B}-3\,aM\,\beta\,\gamma\,n_A\sqrt{aM\,n_A+bM\,n_B}+bM\,\beta\,\gamma\,n_B\sqrt{aM\,n_A+bM\,n_B}\right)<0\,\&\&\,\beta==0.5\,\&\&\,1\geq\gamma>0\,\&\&\,aM==2.\,\&\&\,n_A==2.\,\&\&\,bM==0.5\,\&\&\,n_B==1.0,\ \gamma,\ \text{Reals}\right]$$

$n_A == 2.\ \&\&\ n_B == 1.\ \&\&\ \beta == 0.5\ \&\&\ bM == 0.5\ \&\&\ aM == 2.\ \&\&\ 0.404444 < \gamma \leq 1.$

Thus, the second-order condition for regino A's median resident holds if and only if 0.404444 < $\gamma \leq 1$.

Case (4)

$$\text{Reduce}\left[\frac{1}{4\gamma^3\left(\frac{aM\,n_A}{\gamma}\right)^{3/2}\sqrt{aM\,n_A+bM\,n_B}}\,n_A\left(-4\,aM\,\gamma^2\,n_A\sqrt{\frac{aM\,n_A}{\gamma}}+2\,aM\,\beta\,\gamma^2\,n_A\sqrt{\frac{aM\,n_A}{\gamma}}+aM\,\beta\,n_A\sqrt{aM\,n_A+bM\,n_B}+2\,aM\,\gamma\,n_A\sqrt{aM\,n_A+bM\,n_B}-3\,aM\,\beta\,\gamma\,n_A\sqrt{aM\,n_A+bM\,n_B}+bM\,\beta\,\gamma\,n_B\sqrt{aM\,n_A+bM\,n_B}\right)<0\;\&\&\;\beta==0.5\;\&\&\;1\geq\gamma>0\;\&\&\;aM==2.25\;\&\&\;n_A==2.5\;\&\&\;bM==0.5\;\&\&\;n_B==1.0,\;\gamma,\;\text{Reals}\right]$$

$n_A == 2.5$ && $n_B == 1.$ && $\beta == 0.5$ && bM == 0.5 && aM == 2.25 && 0.395618 < $\gamma \leq 1.$

Thus, the second-order condition for regino A's median resident holds if and only if 0.395618 < $\gamma \leq 1$.

Case (5)

$$\text{Reduce}\left[\frac{1}{4\gamma^3\left(\frac{aM\,n_A}{\gamma}\right)^{3/2}\sqrt{aM\,n_A+bM\,n_B}}\,n_A\left(-4\,aM\,\gamma^2\,n_A\sqrt{\frac{aM\,n_A}{\gamma}}+2\,aM\,\beta\,\gamma^2\,n_A\sqrt{\frac{aM\,n_A}{\gamma}}+aM\,\beta\,n_A\sqrt{aM\,n_A+bM\,n_B}+2\,aM\,\gamma\,n_A\sqrt{aM\,n_A+bM\,n_B}-3\,aM\,\beta\,\gamma\,n_A\sqrt{aM\,n_A+bM\,n_B}+bM\,\beta\,\gamma\,n_B\sqrt{aM\,n_A+bM\,n_B}\right)<0\;\&\&\;\beta==0.5\;\&\&\;1\geq\gamma>0\;\&\&\;aM==2.5\;\&\&\;n_A==3.\;\&\&\;bM==0.5\;\&\&\;n_B==1.0,\;\gamma,\;\text{Reals}\right]$$

$n_A == 3.$ && $n_B == 1.$ && $\beta == 0.5$ && bM == 0.5 && aM == 2.5 && 0.390245 < $\gamma \leq 1.$

Thus, the second-order condition for regino A's median resident holds if and only if 0.390245 < $\gamma \leq 1$.

Case (6)

$$\text{Reduce}\left[\frac{1}{4\gamma^3\left(\frac{aM\,n_A}{\gamma}\right)^{3/2}\sqrt{aM\,n_A+bM\,n_B}}\,n_A\left(-4\,aM\,\gamma^2\,n_A\sqrt{\frac{aM\,n_A}{\gamma}}+2\,aM\,\beta\,\gamma^2\,n_A\sqrt{\frac{aM\,n_A}{\gamma}}+aM\,\beta\,n_A\sqrt{aM\,n_A+bM\,n_B}+2\,aM\,\gamma\,n_A\sqrt{aM\,n_A+bM\,n_B}-3\,aM\,\beta\,\gamma\,n_A\sqrt{aM\,n_A+bM\,n_B}+bM\,\beta\,\gamma\,n_B\sqrt{aM\,n_A+bM\,n_B}\right)<0\;\&\&\;\beta==0.5\;\&\&\;1\geq\gamma>0\;\&\&\;aM==2.75\;\&\&\;n_A==3.5\;\&\&\;bM==0.5\;\&\&\;n_B==1.0,\;\gamma,\;\text{Reals}\right]$$

$n_A == 3.5\ \&\&\ n_B == 1.\ \&\&\ \beta == 0.5\ \&\&\ bM == 0.5\ \&\&\ aM == 2.75\ \&\&\ 0.38671 < \gamma \leq 1.$

Thus, the second-order condition for regino A's median resident holds if and only if $0.38671 < \gamma \leq 1$.

Case (7)

$$\text{Reduce}\left[\frac{1}{4\gamma^3\left(\frac{aM\,n_A}{\gamma}\right)^{3/2}\sqrt{aM\,n_A+bM\,n_B}}\,n_A\left(-4\,aM\,\gamma^2\,n_A\sqrt{\frac{aM\,n_A}{\gamma}}+2\,aM\,\beta\,\gamma^2\,n_A\sqrt{\frac{aM\,n_A}{\gamma}}+aM\,\beta\,n_A\sqrt{aM\,n_A+bM\,n_B}+2\,aM\,\gamma\,n_A\sqrt{aM\,n_A+bM\,n_B}-3\,aM\,\beta\,\gamma\,n_A\sqrt{aM\,n_A+bM\,n_B}+bM\,\beta\,\gamma\,n_B\sqrt{aM\,n_A+bM\,n_B}\right)<0\;\&\&\;\beta==0.5\;\&\&\;1\geq\gamma>0\;\&\&\;aM==3.\;\&\&\;n_A==4.\;\&\&\;bM==0.5\;\&\&\;n_B==1.0,\;\gamma,\;\text{Reals}\right]$$

$n_A == 4.\ \&\&\ n_B == 1.\ \&\&\ \beta == 0.5\ \&\&\ bM == 0.5\ \&\&\ aM == 3.\ \&\&\ 0.384251 < \gamma \leq 1.$

Thus, the second-order condition for regino A's median resident holds if and only if $0.384251 < \gamma \leq 1$.